Practice problems for the second exam

1. A typesetter makes one error in every 500 words typeset. Assume that the probability of an error is the same for each word and an error in one word does not affect the probability of an error in any other word.

1a. What is the probability that there will be no more than one error in one sentence? (A typical sentence contains 20 words.)
1b. Write down, but do **not** evaluate, an expression for the probability of no more than two errors in five pages. (A typical page contains 300 words.)
1c. The exact probability in 1b is 0.42297. Is there an appropriate approximation to this probability? If so, obtain the approximate probability of no more than two errors in five pages.

2. Let Y denote the temperature at which a certain chemical reaction takes place. Suppose that Y has density function given by $f(y) = (4 - y^2)/9$, if $-1 \leq y \leq 2$ and $f(y) = 0$, otherwise.

2a. Suppose this reaction is independently carried out 10 times in the lab and the density function of reaction time is as given above each time. Let $X =$ the number of reactions at which the temperature exceeds 1. What is the distribution of X? (Give the values of any parameters.)
2b. The experimenter decides to carry out the chemical reaction until a temperature above 1 is observed. Again, the reaction is independently carried out and the previous density function for the temperature applies each time. What is the probability that a temperature above 1 will be observed on or before the third reaction?

3. An experiment consists of measuring the reaction time Y (in seconds) to a certain stimulus. The stimulus is such that the reaction time cannot be less than 1 second and more than 3 seconds. The density function for Y is given by $f(y) = cy^{-2}$, if $1 \leq y \leq 3$ and $f(y) = 0$, otherwise.

3a. Find the value of c.
3b. What is the probability that reaction time is at most 2.5 seconds?
3c. Compute the expected reaction time.
3d. If an individual takes more than 1.5 seconds to react, a light comes on and stays on either until one further second has elapsed or until the person reacts (whichever happens first). Determine the expected amount of time that the light remains lit.