Practice problems for the third exam

1. Five automobiles of the same type are to be driven on a 300-mile trip. The first two will use an economy brand of gasoline, and the other three will use a name brand. Let Y_1, Y_2, Y_3, Y_4 and Y_5 be the miles per gallon for the five cars. Suppose these random variables are independent and normally distributed with means $\mu_1 = \mu_2 = 20$ (economy brand), $\mu_3 = \mu_4 = \mu_5 = 21$ (name brand) and variances $\sigma_1^2 = \sigma_2^2 = 4$ (economy brand), $\sigma_3^2 = \sigma_4^2 = \sigma_5^2 = 3.5$ (name brand). Define the random variable D by

$$D = \frac{Y_1 + Y_2}{2} - \frac{Y_3 + Y_4 + Y_5}{3},$$

so that D is a measure of the difference in efficiency between economy gas and name-brand gas.

1a. Obtain $E(D)$, the expected difference in efficiency and $Var(D)$, the variance of the difference in efficiency.

1b. What is the distribution of D and why? Compute $P(D \geq 0)$ and $P(-1 \leq D \leq 1)$.

2. A restaurant serves three fixed-price dinners costing \$7, \$9 and \$10. For a randomly selected couple dining at this restaurant, let X = the cost of the man's dinner and Y = the cost of the woman's dinner. The joint probability distribution of X and Y is given in the following table:

$p(x,y)$		y: 7	9	10
	7	0.05	0.05	0.10
x	9	0.05	0.10	0.35
	10	0	0.20	0.10

2a. Compute the marginal probability distributions of X and Y.

2b. What is the probability that the man's and the woman's dinner cost at most \$9 each?

2c. Find the conditional probability distribution of Y given X = \$9.

2d. Are X and Y independent? Justify your answer.

2e. What is the expected total cost of the dinner for the two people?

2f. Suppose that when a couple opens fortune cookies at the conclusion of the meal, they find the message "You will receive as a refund the difference between the cost of the more expensive and the less expensive meal that you have chosen." How much does the restaurant expect to refund?

3. Consider the Rayleigh distribution with density function

$$f(y;\theta) = \frac{y}{\theta} e^{-\frac{y^2}{2\theta}}, y > 0, \theta > 0.$$

3a. Find the hazard rate corresponding to the Rayleigh distribution. How is the value of the hazard rate function affected as (time) y increases?

3b. Assume that the useful lifetimes X and Y (in years) of two machine components that function independently can be modelled using the Rayleigh distribution with parameters $\theta = 1$ and 2, respectively. What is the joint density of X and Y? What is the probability that the lifetime of at least one component exceeds 1 year?