STA 205 - Probability and Measure Theory

Homework 3

1. Consider an increasing sequence $\{X_n\}_{n=1}^{\infty}$ of $\overline{R}$-valued random variables defined on a common probability space $(\Omega, \mathcal{F}, \mathcal{P})$. For each $\omega \in \Omega$, set $X(\omega) = lim_{n\to\infty} X_n(\omega)$. If $E(X_1) > -\infty$, then $lim_{n\to\infty} E(X_n) = E(X)$.

2. Let Y be an R-valued random variable, defined on some probability space $(\Omega, \mathcal{F}, \mathcal{P})$, for which $E(|Y|) < \infty$. Prove that $lim_{c\to\infty} E(|Y|\, 1_B) = 0$, where 1_B is the indicator function of the event $B = \{\omega \in \Omega : |Y(\omega)| \geq c\}$.

3. Consider a sequence $\{X_n\}_{n=1}^{\infty}$ of $\overline{R}$-valued random variables, defined on a common probability space $(\Omega, \mathcal{F}, \mathcal{P})$, which converges almost surely to a random variable X. Assume that $sup_n E(X_n^2) < \infty$. Show that $E(X^2) < \infty$.