STA-226, Spring Problem set 1. Due Thursday Jan 25th.

1. This example will help you review the basic ideas discussed in Lecture 1. It is taken from the classic book of Chernoff and Moses (1959).

"Jane Smith can cook spaghetti, hamburger or steak for dinner. She has learned from past experience that if her husband is in a good mood she can serve him spaghetti and save money, but if he is in a bad mood only a juicy steak can calm him down and make him bearable. In short, there are three actions:

a_1 prepare spaghetti;
a_2 prepare hamburger;
a_3 prepare steak.

Three states of nature:

θ_1 Mr. Smith is in a good mood;
θ_2 Mr. Smith is in a normal mood;
θ_3 Mr. Smith is in a bad mood.

The loss table is

	a_1	a_2	a_3
θ_1	0	2	4
θ_2	5	3	5
θ_3	10	9	6

The experiment she performs is to tell him when he returns home that she lost the afternoon paper. She foresees 4 possible responses. These are:

x_1 "Newspaper will get lost";
x_2 "I keep telling you 'a place for everything and everything in its place"';
x_3 "Why did I ever get married?".
x_4 an absent-minded far-away look.

The frequency table of responses is

	x_1	x_2	x_3	x_4
θ_1	0.5	0.4	0.1	0
θ_2	0.2	0.5	0.2	0.1
θ_3	0	0.2	0.5	0.3

(a) List four strategies and evaluate their average losses.

(b) Point out the minimax average loss strategy

(c) Which is the best strategy if Mr. Smith is in a good mood 30% of the time and in a normal mood 50% of the time?"

Notes: In (a) and (b) average refers to the outcome of the experiment; so in (b) "average loss" is what we called risk; in (b) and (c) restrict attention to the the four strategies that you have chosen. In (c) "best" refers to the Bayes strategy.

Additional question: Work out the regret table and find the minimax regret strategy among the four that you have selected.

2. In order to choose an action a based on the loss $L(\theta, a)$, you elicit the opinion of two experts about the probabilities of the various outcomes of θ (you can assume if you wish that θ is a discrete random variable). The two experts give you distributions π_1 and π_2. Suppose that the action a^* is Bayes for both distributions π_1 and π_2. Is it true that a^* must be Bayes for all weighted averages of π_1 and π_2, that is for all the distributions of the form $\alpha\pi_1 + (1-\alpha)\pi_2$, with $0 < \alpha < 1$?

3. From Lindley (1985). "At British parliamentary elections a firm of bookmakers was offering odds of 4–7 against labour and 5–4 against the conservatives. Equating utility with money, show that one should bet unless one's personal assessment of Labour's chances of winning lies between .56 and .64"

Note: Assume that there are only two parties. "Odds against labour" means that the bookmakers are posting odds of 4:7 for bets on the event that labor loses the election.

4. Next week the Duke soccer team plays American University and Akron. Call E the event *Duke beats Akron* and F the event *Duke beats American*. Suppose a book maker is posting the following betting odds and is willing to take stakes of either sign on any combination of the events:

Event	Odds
E	$4:1$
F	$3:2$
$E \cup F$	$9:1$
$E \cap F$	$2:3$

Find stakes that will enable you to win money from the book maker no matter what the results of the games are. You can use Splus or Mathematica if you like.