STA-226, Spring Problem set 6. Due Thursday March 1st.

1. Four well-shuffled cards, the kings of clubs, diamonds, hearts and spades, are laid face down on the table. A man is offered the following choice. Either: A randomly selected card will be turned over. If it is red, he will win £100; if it is black, he will lose £100; Or: A randomly selected card will be turned over. He may then choose to pay £35 and call the bet off or he may continue. If he continues, one of the remaining three cards will be randomly selected and turned over. If it is red, he will win £100; if it is black, he will lose £100. Draw a decision tree for the problem. If the decision maker is risk neutral, what should he do? Suppose instead that his utility function for sums of money in this range is $u(x) = \log(1 + x/200)$. What should he do in this case?

2. A builder is offered two plots of land for house building at £20,000 each. If the land does not suffer from subsidence, then he would expect to make £10,000 net profit on each plot, when a house is built on it. However, if there is subsidence, the land is only worth £2000 so he would make an £18,000 loss. He believes that the chance that both plots will suffer fromsubsidence is 0.2, the chance that one will suffer from subsidence is 0.3, and the chance that neither will is 0.5. He has to decide whether to buy the two plots or not. Alternatively, he may buy one, test it for subsidence, and then decide whether to buy the other plot. Assuming that the test is a perfect predictor of subsidence and that it costs £200 for the test, what should he do? Assume that his preferences are determined purely by money and that he is risk neutral.

3. You have decided to buy a car, and have eliminated possibilities until you are left with a straight choice between a brand new car costing £4000 and a second-hand car costing £2750. You must have a car regularly for work. So, if the car that you buy proves unreliable, you will have to hire a car while it is being repaired. The guarantee on the new car covers both repair costs and hire charges for a period of two years. There is no guarantee on the second-hand car. You have considered the second-hand-car market and noticed that cars tend to be either very good buys or very bad buys, few cars falling between the two extremes. With this in mind you consider only two possibilities.

(i) The second-hand car is a very good buy and will involve you in £750 expenditure on car repairs and car hire over the next two years.

(ii) The second-hand car is a very bad buy and will cost you £1750 over the next two years.

You also believe that the probability of it being a good buy is only .25. However, you may ask the AA for advice at negligible financial cost, but risking a probability of .3 that the second-hand car will be sold while they are arranging a road test. The road test will give a satisfactory or unsatisfactory report with the following probabilities.

	Conditional on the car being:	
Probability of AA report being:	a very bad buy	a very good buy
satisfactory	.1	.9
unsatisfactory	.9	.1

If the second-hand car is sold before you buy it, you must buy the new car. Alternatively you may ask your own garage to examine the second-hand car. They can do so immediately, thus not risking the loss of the option to buy, and will also do so at negligible cost, but you do have to trust them not to take a 'back-hander' from the second-hand-car salesman. As a result, you evaluate thir reliability as:

	Conditional on the car being:	
Probability of garage report being:	a very bad buy	a very good buy
satisfactory	.5	1
unsatisfactory	.5	0

You can arrange at most one of the tests. What should you do, assuming that you are risk neutral for monetary gains and losses and that no other attribute is of importance to you?

4. Suggest at least one good way to improve the functions in the `splustree.s` file, based on trying them in one of problems 1–3.