STA-226, Spring Problem set 8. Due Thursday March 22nd.

1. Using a simulation, approximate the distribution of $\mathcal{I}(\pi \to \pi_x)$ in the example in section 16.4. Suppose each observation costs \$230. Compute the marginal cost-effectiveness ratio for performing the experiment in the example (vs the status quo of no experimentation). Use the negative of the loss as the measure of effectiveness.

2. In the context of Chapter 17, prove that if the statistic t is sufficient for x in the Neyman-Fisher sense (that is if $p(x|\theta, t)$ is independent of θ), then $\mathcal{I}(\mathcal{E}_x) = \mathcal{I}(\mathcal{E}_t)$. Here $\mathcal{E}_x$ adn $\mathcal{E}_t$ are the experiment that consist of observing x and t respectively.

If you are stuck, go hunt down the factorization theorem about sufficient statistics.

3. Do you think the result of problem 2 would also apply to the expected information in the context of Chapter 16? Can you prove why or why not?

4. Consider an experiment $\mathcal{E}$ that consists of observing n conditionally independent random variables $x_1, \ldots, x_n$, with $x_i \sim N(\theta, \sigma^2)$. Suppose also that a priori $\theta \sim N(\mu, \tau^2)$. Show that

$$\mathcal{I}(\mathcal{E}) = \frac{1}{2} \log \left(1 + n \frac{\tau^2}{\sigma^2} \right).$$

You can use facts about conjuagte priors from Bernardo and Smith, or Berger. However, please rederive $\mathcal{I}$.