1. The annual wages, excluding board, of U.S. farm laborers in 1926 were normally distributed with an average of \$586 and a standard deviation of \$97.

(a) (3 pts) How many standard deviations away from the average is a 1926 U.S. laborer's annual wage of \$780?

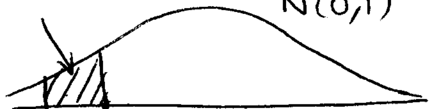
$$z = \frac{780-586}{97} = 2$$

(b) (5 pts) In 1926 what percentage of U.S. farm laborers had an annual wage between \$360 and \$420?

$$\frac{360-586}{97} = -2.33 \qquad \frac{420-586}{97} = -1.71$$

area $= 0.0436 - 0.0099 = 0.0337 = 3.37\%$

N(0,1)



-2.33 -1.71

(c) (6 pts) Find the 85th percentile of the annual wage of U.S. farm laborers in 1926.

area $= 85\%$

area $= 15\%$

$z = 1.04$

The 85th percentile of a N(0,1) distribution is 1.04

The 85th percentile of the annual wage of US farm laborers in 1926 was $586 + 97 \cdot 1.04 = \$686.88$

2. (8 pts) A stock market analyst examined the prospect of the shares of a large number of corporations. When the performance of these stocks was investigated one year later, it turned out that 25% performed much better than the market average, 25% much worse, and the remaining 50% about the same as the average. Forty percent of the stocks that turned out to do much better than the market were rated "good buys" by the analyst, as were 20% of those that did as well as the market and 10% of those that did much worse. What is the probability that a randomly chosen stock rated a "good buy" by the analyst performed much better than the market average?

A = about the average $\qquad P(A) = .5 \qquad P(G|A) = .2$

B = better than the average $\qquad P(B) = .25 \qquad P(G|B) = .4$

W = worse than the average $\qquad P(W) = .25 \qquad P(G|W) = .1$

G = rated "good buy"

$$P(B|G) = \frac{P(G|B)\cdot P(B)}{P(G|A)\cdot P(A) + P(G|B)\cdot P(B) + P(G|W)\cdot P(W)}$$

$$= \frac{.4 \cdot .25}{.2 \cdot .5 + .4 \cdot .25 + .1 \cdot .25} = \frac{.1}{.1 + .1 + .025} = \frac{.1}{.225} = \frac{4}{9} \approx .44$$

3. A student has not studied for his multiple choice quiz. The quiz has 10 questions. Among the possible answers for each question, exactly one is correct. The student randomly chooses one answer for each question. His choice for one question is independent of his choice for the other questions.

(a) (5 pts) If each question has 5 possible answers, what is the probability that the student answers exactly three questions correctly?

X = number of questions the student answers correctly.

$X \sim \text{Binomial}(10, .2)$

$P(X=3) = \binom{10}{3}(.2)^3(.8)^7$

$\approx .2013$

(b) (8 pts) If each of the first eight questions have 5 possible answers and the other two questions have 4 possible answers, what is the probability that the student answers exactly one question correctly?

P(1 correct answer)

= P(1 correct answer from the first 8 questions and 0 correct answers from the last 2 questions)

+ P(0 correct answer from the first 8 questions and 1 correct answer from the last 2 questions)

$\approx \binom{8}{1}(.2)^1(.8)^7 \cdot \binom{2}{0}(.25)^0(.75)^2 + \binom{8}{0}(.2)^0(.8)^7 \cdot \binom{2}{1}(.25)^1(.75)^1$

$\approx .25166$

4. (7 pts) Is the following function g a valid p.d.f. (probability density function)?

$$g(x) = \begin{cases} \frac{60}{7}x(\frac{9}{10} - x) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

It yes, show that it is a valid p.d.f. If no, explain why not valid.

Note that $g(x) < 0$ if x is in $(\frac{9}{10}, 1]$ (i.e. if $\frac{9}{10} < x \le 1$).

Therefore g is not a valid p.d.f.

5. A box contains four tickets numbered 1, 1, 2 and 3, respectively. Another box contains three tickets numbered 1, 2 and 2, respectively. One ticket will be randomly drawn from each of these two boxes.

(a) (5 pts) Write down the sample space for this experiment.

Each sample point will be of the form (draw from the first box, draw from the second box)

The sample space is the set of sample points.

Sample points	Probability
(1,1)	1/6
(1,2)	1/3
(2,1)	1/12
(2,2)	1/6
(3,1)	1/12
(3,2)	1/6

(b) (5 pts) Find the probability that both tickets drawn will have the same number on them.

P(both tickets will have the same number)

$= P\{(1,1)\} + P\{(2,2)\} = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

(c) (7 pts) Define a random variable X as the sum of the numbers on the tickets drawn. Write the distribution of X by giving the p.m.f. (probability mass function).

X can take values 2, 3, 4, 5

sample point	X	probability
(1,1)	2	1/6
(1,2)	3	1/3
(2,1)	3	1/12
(2,2)	4	1/6
(3,1)	4	1/12
(3,2)	5	1/6

~~P(X)~~

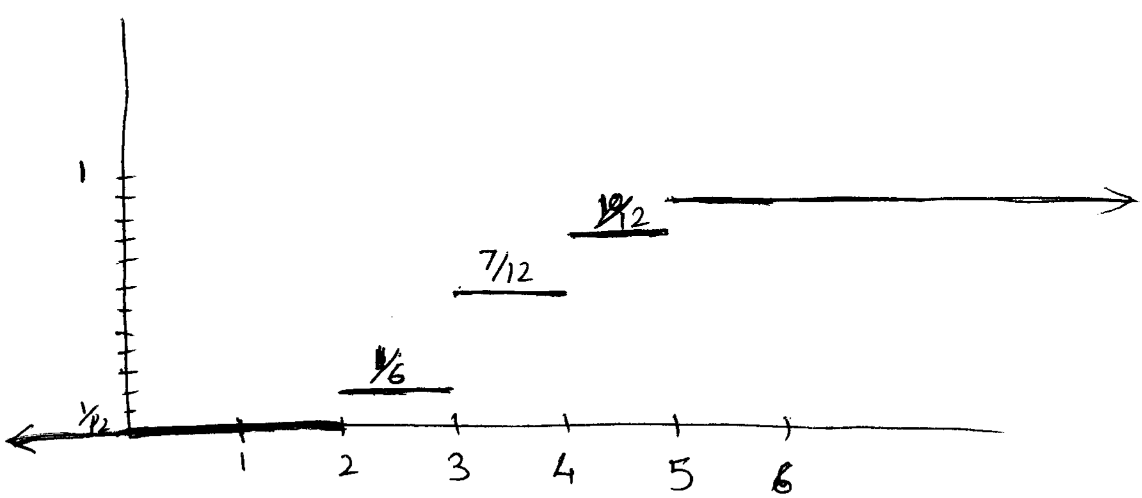
$P(X=2) = \frac{1}{6}$

$P(X=3) = \frac{1}{3} + \frac{1}{12} = \frac{5}{12}$

$P(X=4) = \frac{1}{6} + \frac{1}{12} = \frac{1}{4}$

$P(X=5) = \frac{1}{6}$

(d) (7 pts) Draw the c.d.f. (cumulative distribution function) of X. Mark each axis with appropriate scales.



6. The probability that it will rain on Monday is .5, the probability that it will rain on Tuesday is .4, the probability that it will rain at least on one of Monday and Tuesday is .8.

(a) (5 pts) Find the probability that it will rain on both Monday and Tuesday.

Let M be the event that it will rain on Monday and T be the event that it will rain on Tuesday

$P(M) = .5, \quad P(T) = .4, \quad P(M \cup T) = .8$

$$P(M \cup T) = P(M) + P(T) - P(M \cap T)$$

$$.8 = .5 + .4 - P(M \cap T)$$

So, $P(M \cap T) = -.8 + .5 + .4 = .1$

(b) (3 pts) Are the events of raining on Monday and raining on Tuesday independent? Justify your answer.

$P(M \cap T) = .1, \quad P(M) \cdot P(T) = .4 \cdot .5 = .2$

Therefore, $P(M \cap T) \neq P(M) \cdot P(T)$

So, M and T are not independent.

(c) (5 pts) What is the probability that it will rain on both days given that it will rain on at least one of those two days?

$$P(M \cap T \mid M \cup T) = \frac{P[(M \cap T) \cap (M \cup T)]}{P(M \cup T)}$$

$$= \frac{P(M \cap T)}{P(M \cup T)}$$

Since $M \cap T \subset M \cup T$, and hence $(M \cap T) \cap (M \cup T) = M \cap T$

$$= \frac{.1}{.8} = .125$$

7. The relative humidity X, measured at a particular location over days, has a p.d.f. given by

$$f(x) = \begin{cases} cx(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

where c is a positive constant.

(a) (5 pts) Find the value of c.

$$1 = \int_0^1 c x (1-x)\, dx = c \int_0^1 (x - x^2)\, dx = c \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= c\left(\frac{1}{2} - \frac{1}{3}\right) = c \cdot \frac{1}{6}$$

So, $c = c$

(b) (7 pts) Find the c.d.f. of X.

$$\int_0^y 6x(1-x)\,dx = 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^y = 6\left(\frac{y^2}{2} - \frac{y^3}{3}\right)$$
$$= 3y^2 - 2y^3$$

The c.d.f. of X is

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 3x^2 - 2x^3 & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$$

(c) (9 pts) Find the variance of X.

$$V(X) = E(X^2) - (E(X))^2$$

$$E(X) = \int_0^1 x \cdot 6x(1-x)\,dx = 6\int_0^1 (x^2 - x^3)\,dx$$
$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 6\left(\frac{1}{3} - \frac{1}{4}\right) = 6 \cdot \frac{1}{12} = \frac{1}{2}$$

$$E(X^2) = \int_0^1 x^2 \cdot 6x(1-x)\,dx = 6\int_0^1 (x^3 - x^4)\,dx$$
$$= 6\left[\frac{x^4}{4} - \frac{x^5}{5}\right]_0^1 = 6\left(\frac{1}{4} - \frac{1}{5}\right) = \frac{6}{20} = \frac{3}{10}$$

$$V(X) = E(X^2) - (E(X))^2$$
$$= \frac{3}{10} - \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{20}$$
$$= .05$$