Homework 10 Solutions

1 Exercise 11.2

We have $\sum_{i=1}^{n} x_i = 720$, $\sum_{i=1}^{n} y_i^2 = 105817$, $\sum_{i=1}^{n} y_i = 721$, $\sum_{i=1}^{n} x_i y_i = 106155$, $\sum_{i=1}^{n} x_i^2 = 106554$, n=10, $S_{xy} = 54243$
Then $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = 0.9913916$. and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 0.7198048$. $S_{xx} = 54714$.

2 Exercise 11.4

We get $\sum_{i=1}^{n} x_i = 155.05$, $\sum_{i=1}^{n} y_i^2 = 1993.8156$, $\sum_{i=1}^{n} y_i = 94.48$, $\sum_{i=1}^{n} x_i y_i = 3011.3709$, $\sum_{i=1}^{n} x_i^2 = 4763.979$, n=10, $S_{xy} = 1546.459$, $S_{xx} = 2359.929$. As in exercise 11.2, we get $\hat{\beta}_1 = 0.65330$ and $\hat{\beta}_0 = -0.7124$. The least square line is given by $\hat{y} = -0.712 + 0.655x$. When x=12, $\hat{y} = 7.15$

3 Exercise 11.14a

Fom exercise 11.4, $S_{yy} = \sum_{i=1}^{n} (y_i - \bar{y})^2 = 1101.1686$, $S_{xy} = 1546.459$ and SSE=1101.1686-(0.6552528)(1546.553) = 87.84701. And $s^2 = \frac{SSE}{8}$.

4 Exercise 11.22

- $\hat{\beta}_1 = 0.118$ and $\hat{\beta}_0 = 0.72$
- SSE=0.125 and $s^2 = \frac{SSE}{n-2} = 0.013$. A 95 % Confidence interval for β_1 is $[0.118 + / - 2.776\sqrt{0.013}\sqrt{0.00059}]$.
- We must test $H_0 : \beta_0 = 0$ vs $H_0 : \beta_0 \neq 0$. The test statistic is :
$t = \frac{\hat{\beta}_0}{s\sqrt{c_{00}}} = 4.587$. Since the p-value is smaller than 0.05, then we reject H_0 at the $\alpha = 0.05$ level.

5 Exercise 11.26a

We test $H_0 : \beta_1 = 0$ vs $H_0 : \beta_1 \neq 0$. Construct $t = \frac{\hat{\beta}_1 - 0}{\sqrt{V(\hat{\beta}_1)}}$ which for small particle catalyst gives $t_1 = 7.67$ and large $t_2 = 9.84$. The rejection region for $\alpha = 0.05$ is t bigger than $t_{0.025,29} = 2.045$ in absolute value for the first experiment and t bigger than $t_{0.025,9} = 2.262$ in absolute value for the second experiment. In case the null hypothesis is rejected we conclude that the slopes are significantly different from zero.

6 Exercise 11.33

From exercises 11.4 and 11.14, we got results for $\hat{y}$, x, s^2 and S_{xy}. With the results from exercise 11.31, we get the 95% confidence interval which is:
$7.15 +/- 2.48$.

7 Exercise 11.41

When x=12, $\hat{y} = 7.15$ and the 95% prediction interval is:
$7.15 +/- 2.306\sqrt{s^2(1 + \frac{1}{10} + \frac{(12-15.504)^2}{2359.929})}$ which gives $[-0.86; 15.18]$.