Homework Solutions 2

1 Exercise 2.16

a. The sample points are (HH, TH, HT, TT).

b. Assuming the coins are balanced, each sample point is equally probable. The common probability is $\frac{1}{4}$.

c. A=(HT, TH) and B=(HT, TH, HH).

d. Using parts b and c, we get P(A)=0.25+0.25=0.5 and P(B)=0.75, P(A∩B)=P(A)=0.5. P(A∪B)=P(B)=0.75 and P($\bar{A} \cup B$)=P(S)=1.

2 Exercise 2.17

a. Here the order is important since (V_1, V_2) is different from (V_2, V_1). The sample points are of the form (V_i, V_j) for i,j=1,2,3.

b. All the points have equal prob of $\frac{1}{9}$.

c. A=(same vendor gets both) = $\{(V_1, V_1), (V_2, V_2), (V_3, V_3)\}$ and B = (vendor gets at least one) = $(V_1, V_2), (V_2, V_1), (V_2, V_3), (V_3, V_2), (V_2, V_2)$ then P(A)= 0.33, P(B)= $\frac{5}{9}$ and P(A∪B) = $\frac{7}{9}$ and P(A∩B) = P(V_2, V_2)=$\frac{1}{9}$.

3 Exercise 2.18

a. Let N_1 and N_2 be empty cans and W_1 and W_2 be the cans filled with water. Then the possible pairs are $(N_i W_j)$ along with $(N_1 N_2)$ for i,j=1,2.

b. If rod worthless, then expert is guessing and probability is $\frac{1}{6}$. and probability that each experts picks the two cans containing water is $\frac{1}{6}$.

4 Exercise 2.20

a. and b. Here order is unimportant . There are six possible outcomes, each with probability $\frac{1}{6}$.

c. P(minority hired)=0.5

5 Exercise 2.25

a. Define the events E=familiy's income exceeds 35353 dollars and N=family's income does not exceed 35353 dollars. One of the possible sample points would be $E_1 = (EEEE)$. There are 16 of course.

b. $A = (E_1, E_2, ..., E_{11})$
$B = (E_6, E_7, ..., E_{11})$ and $C = (E_2, E_3, ..., E_5)$.

c. By the definition of median each event is equally likely and P(E_i)=$\frac{1}{16}$, then P(A)=$\frac{11}{16}$ and P(B)=$\frac{3}{8}$ and P(C)=$\frac{1}{4}$.

6 Exercise 2.31

a. Use mn rule.The first die has six possible results and the second has six so a total of 36 possible sample points.

b. Let A=observe a sum of 7 on the two dice. There are six sample points that satisfy this and thus we get P(A)=P(observe a sum of 7)=$\frac{6}{36}$.

7 Exercise 2.39

a.

$$\binom{130}{2} = 8385$$

.

b. 26*26=676 two letter codes.
26*26*26=17576 three letter codes
=18252 total major codes available.

c. 8385+130=8515 required

d. yes

8 Exercise 2.41

There are

$$\binom{50}{3} = 19600$$

ways to choose the three winners. Each way is equally probable..

a. There are

$$\binom{4}{3} = 4$$

ways for the organizers to win all the prizes. Hence the probability is $\frac{4}{19600}$.

b. The organizers can win exactly two of the prizes if one of the other 46 people wins one prize. Using the mn rule, there are 276 ways

$$\binom{4}{2} * \binom{46}{1} = 276$$

ways for this to occur. Hence the probability is $\frac{276}{19600}$.

c.

$$\binom{4}{1} * \binom{46}{2} = 4140$$

. The prob is $\frac{4140}{19600}$.

d.

$$\binom{46}{3} = 15180$$

. The prob is $\frac{15180}{19600}$.

9 Exercise 2.50

$6!(\frac{1}{6})^6 = \frac{5}{324}$.

10 Exercise 2.51

$5!(\frac{2}{6})(\frac{1}{6})^4=\frac{5}{162}$.

11 Exercise 2.60

Define the events U=job is unsatisfactory and A=plumber A does the job. It is given that P(A)=0.4, P(U)=0.1, P(A|U)=0.5

a. The probability of interest is P(U|A)=0.125

b. P($\bar{U}$ | A)=0.875

12 Exercise 2.61

a. 0.40

b. 0.37

c. 0.1

d. 0.67

e. 0.60

f. 0.33

g. 0.90

h. 0.27

i. 0.25

13 Exercise 2.62

a. since P(A|B)=P(A), thus P(B|A)=P(B) by the formula of conditional prob.

b. If P(B|A)=P(B), then similarily, we have P(A|B)=P(A).

c. Assume P(A∩B)=P(A)P(B), then the results follows from the ones above.

14 Exercise 2.70

A=device A detects smoke
B=device B detects smoke

a. P(A∪B)=0.97

b. P(smoke undetected)=1-P(A∪B)=0.03

15 Exercise 2.84

Applying result of exercise 2.80, let U=$A \cup B$ and V=C, then we have P($A \cup B \cup C$)≥1-P($\bar{A}$)-P($\bar{B}$)-P($\bar{C}$)

16 Exercise 2.94

If the victim is to be saved, a proper donor must be found within eight minutes allowing 2 minutes for transfer of blood. Thus 4 people can be typed and the patient will be saved if a proper donor is found on the first, second, third, or fourth try. Note that P(A)=0.4, where A is the event that an a type A, Rh-positive donor is found. Then P(saving the patient)=P(A on first trial, or first A on second, or first A on third or first A on fourth), thus P(saving patient)=$0.4+0.6*0.4+(0.6)^2 0.4+(0.6)^3 0.4=0.8704$

17 Exercise 2.97

a. $\frac{1}{n}$

b. $(\frac{n-1}{n})(\frac{1}{n-1}) = \frac{1}{n}$ second try
$(\frac{n-1}{n})(\frac{n-2}{n-1})(\frac{1}{n-2}) = \frac{1}{n}$ third try

c. P(gain access)=$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{3}{7}$

18 Exercise 2.104

C=contract lung cancer and S=worked in a shipyard. Then P($S \mid C$)=0.22 and P($S \mid \bar{C}$)=0.14. Also P(C)=0.0004. Using Bayes rule, we get P($C \mid S$)=0.0006

19 Exercise 2.114

A=woman's name selected from list 1 and B=woman's name selected from list 2. Then P(A)=$\frac{5}{7}$, P($\bar{B} \mid A$)=$\frac{6}{9}$ and P($\bar{B} \mid \bar{A}$)=$\frac{7}{9}$, now by Bayes rule, we get P($A \mid \bar{B}$)=$\frac{30}{44}$.

20 Exercise 2.120

Let Y=number of positions the spinner did not land on; Y=2,3
P(Y=2)=$\frac{3}{4}$
P(Y=3)=$\frac{1}{4}$.