$-\quad B-A \quad -\quad B-A \quad 4$

4.30 Let X = # of parachutists that land past the midpoint.

$$p = \text{probability that one lands past the midpoint} = \int_{(A+B)/2}^{B} \frac{1}{B-A}\,dy$$

$$= \left(\frac{1}{B-A}\right) y\Big]_{(A+B)/2}^{B} = \frac{1}{2}$$

X is binomial with $n = 3$, $p = \frac{1}{2}$.

$$P(X = 1) = 3\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^2 = \tfrac{3}{8}$$

4.46 The next few exercises are designed to provide practice for the student in evaluating areas under the normal curve. The following notes may be of some assistance.

(1) Table 4 tabulates the area under the standard normal curve to the right of a specified value z_0. See Figure 4.11. Denote the area obtained by indexing $z = z_0$ in Table 4 by $A(z_0)$ and the desired area by A.

(2) Because of the symmetry of the normal distribution, and since the total area under the curve is 1, the total area lying on one side of 0 will be .5. Thus in order to calculate the area between 0 and z_0 (when $z_0 > 0$) we index z_0, which gives us $A(z_0)$. We then subtract $A(z_0)$ from .5. That is, $A = .5 - A(z_0)$.

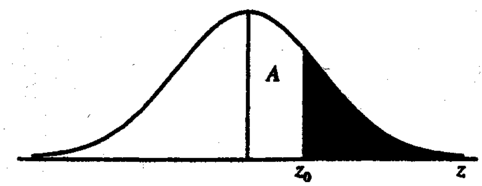

Figure 4.11

(3) Notice that Z is actually a random variable that may take on an infinite number of values, both positive and negative. However, since the standardized normal curve is symmetric about 0, a left-hand area (i.e., an area corresponding to a negative value of z) may be evaluated by indexing the corresponding positive value in Table 4.

(a) The area between $z = 0$ and $z = 1.2$ is $A_1 = .5 - A(1.2) = .5 - .1151 = .3849$. See Figure 4.12.

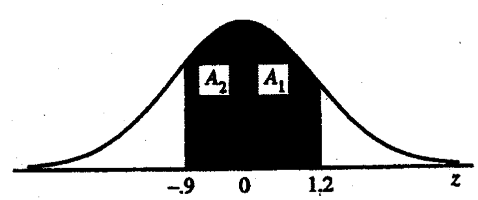

Figure 4.12

(b) The area between $z = 0$ and $z = -.9$ is $A_2 = .5 - A(-.9) = .5 - A(.9)$ $.5 - .1841 = .3159$.

(c) The desired area is A_1, as shown in Figure 4.13. Note that $A(.3) = .3821$ and $A(1.56) = .0594$. $A_1 = A(.3) - A(1.56) = .3821 - .0594 = .3227$.

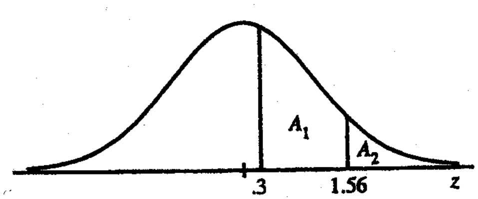

Figure 4.13

(d) The desired area is $A_1 + A_2$ $= .5 - A(-.2) + .5 - A(.2)$ $= 1 - 2(.4207) = .1586$. See Figure 4.14.

(e) The desired area is $A(-.2) - A(-1.56)$ $= .4207 - .0594 = .3613$.

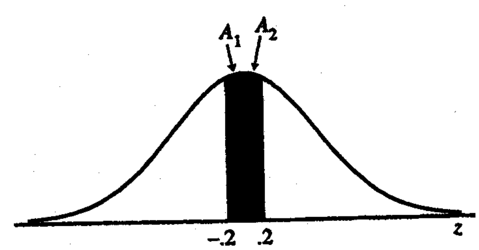

Figure 4.14

4.52 The random variable Y (bearing diameter) is normally distributed with $\mu = 3.0005$ and $\sigma = .0010$. Since "scrap" bearings are those whose diameters are greater than 3.002 or less than 2.998, the necessary area is $P(Y > 3.002) = P(Y < 2.998)$. Corresponding z values are

$$z_1 = \frac{3.002-3.0005}{.0010} = 1.5 \quad \text{and} \quad z_2 = \frac{2.998-3.0005}{.0010} = -2.5$$

and the necessary fraction is

$$P(Y > 3.002) + P(Y < 2.998) = P(Z > 1.5) + P(Z < -2.5) = A(1.5) + A(-2.5)$$
$$= .0668 + .0062 = .0730.$$

4.60 Let Y denote the variable "exam score."

a. $P(Y > 72) = P\left(Z > \frac{72-78}{6}\right) = P(Z > -1) = 1 - P(Z < -1) = 1 - P(Z > 1)$
$= 1 - A(1) = .8413$

b. We seek c such that $P(Y > c) = .1$. Now
$$.1 = P(Y > c) = P\left(Z > \frac{c-78}{6}\right) = A\left(\frac{c-78}{6}\right)$$
The value of z_0 such that $A(z_0) = .1$ is $z_0 = 1.28$. So it must be that
$$1.28 = \frac{c-78}{6}$$
and
$$c = 6(1.28) + 78 = 85.68$$

c. We seek c such that $P(Y > c) = .281$. So
$$.281 = P(Y > c) = P\left(Z > \frac{c-78}{6}\right) = A\left(\frac{c-78}{6}\right)$$
which implies that $\frac{c-78}{6} = .58$ and $c = 81.48$.

d. The score that cuts off the lowest 25% is the score c such that
$$P\left(Z < \frac{c-78}{6}\right) = .25$$
Now, $P(Z > .67) = .25$ so
$$P(Z < -.67) = .25.$$
Hence
$$\frac{c-78}{6} = -.67$$
and
$$c = (-.67)(6) + 78 = 73.98$$
We must now find $P(Y > 73.95 + 5)$. This probability is
$$P(Y > 78.95) = P\left(Z > \frac{78.95-78}{6}\right)$$
$$= P(Z > .16) = .4364$$

e. $P(Y > 84|Y > 72) = \frac{P(Y>84)}{P(Y>72)} = \frac{P\left(Z>\frac{84-78}{6}\right)}{P\left(Z>\frac{72-78}{6}\right)} = \frac{P(Z>1)}{P(Z>-1)} = \frac{P(Z>1)}{1-P(Z>1)}$
$= \frac{.1587}{.8413} = .1886$

4.66 $A = L \times W = |Y| \times 3|Y| = 3Y^2$
$$E(A) = E(3Y^2) = 3E(Y^2) = 3(V(Y) + \mu^2) = 3(\sigma^2 + \mu^2)$$