7.3 Since the distribution of basal areas is normally distributed with mean μ and variance $\sigma^2 = 16$, the sample mean will also be normally distributed, from Theorem 7.1. Then

$$P\left(|\overline{Y}-\mu| \le 2\right) = P\left[-2 \le (\overline{Y}-\mu) \le 2\right] = P\left(\frac{-2}{\left(\frac{\sigma}{\sqrt{n}}\right)} \le \frac{Y-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)} \le \frac{2}{\left(\frac{\sigma}{\sqrt{n}}\right)}\right)$$
$$= P(-1.5 \le z \le 1.5)$$
$$= 1 - 2P(Z > 1.5) = 1 - 2(.0668) = .8664$$

7.4 Refer to Exercise 7.3. It is necessary to have

$$P\left(|\overline{Y}-\mu \le 1|\right) = .90 \qquad \text{or} \qquad P\left(\tfrac{-\sqrt{n}}{4} \le z \le \tfrac{\sqrt{n}}{4}\right) = .90$$

The inequality will be satisfied if we take $\frac{\sqrt{n}}{4} = 1.645$, or $n = 43.30$. Hence 44 trees must be sampled.

7.6 Similar to Exercise 7.4. It is necessary to have

$$P\left(|\overline{Y}-\mu| \le .5\right) = .95 \qquad \text{or} \qquad P\left(|z| \le \tfrac{.5\sqrt{n}}{\sqrt{.4}}\right) = .95$$

That is, $\frac{.5\sqrt{n}}{\sqrt{.4}} = 1.96$ or $n = 6.15$. Thus at least 7 tests must be run.

7.22 **a.** $P(\overline{X} > 4.5) = P\left(\frac{\overline{X}-4}{2/\sqrt{100}} > \frac{4.5-4}{2/\sqrt{100})}\right) \approx P(Z > 2.5) = .0062.$

b. There are several correct answers to this problem. We choose the answer that produces an interval symmetric about μ. That is we want to find an a so that

$$P(\mu - a < \overline{X} < \mu + a) = P(|\overline{X}-\mu| > a) = .95.$$

Note that

$$P(|\overline{X}-\mu| > a) = P\left(\left|\tfrac{\overline{X}-\mu}{\sigma/\sqrt{n}}\right| > \tfrac{a}{\sigma/\sqrt{n}}\right) \approx P\left(|Z| > \tfrac{a}{\sigma/\sqrt{n}}\right).$$

where Z is a standard normal random variable. Then setting $\frac{a}{\sigma/\sqrt{n}} = 1.96$, or $a = 1.96\,\sigma/\sqrt{n}$, will give an approximate 95% interval. Therefore our interval is $\mu \pm 1.96\ \sigma/\sqrt{n}$. For this particular problem we get the interval $14 \pm 1.96(2)/\sqrt{100}$, (13.608, 14.392).

7.23 The Central Limit Theorem (Theorem 7.4) states that $Y_n = \frac{\sqrt{n}(\overline{X}-\mu)}{\sigma}$ converges in distribution to a standard normal random variable, which is denoted by Z. For this exercise, $n = 100$, $\sigma = 2.5$, and the approximation is

$$P\left(|\overline{X}-\mu| \le .5\right) = P\left(-.5 \le \overline{X}-\mu \le .5\right) = P\left[\tfrac{-.5(10)}{2.5} \le Z \le \tfrac{.5(10)}{2.5}\right]$$
$$= P(-2 \le Z \le 2) = 1 - 2(.0228) = .9544$$

7.24 Refer to Exercise 7.23. It is now necessary to choose n such that

$$P\left(|\overline{X}-\mu| \le .4\right) = .95$$

or

$$P\left(-.4 \le \overline{X}-\mu \le .4\right) = P\left(\tfrac{-.4\sqrt{n}}{2.5} \le Z \le \tfrac{.4\sqrt{n}}{2.5}\right) = .95.$$

This probability statement will be satisfied by taking $\frac{.4\sqrt{n}}{2.5} = 1.96$, which implies $\sqrt{n} = \frac{4.9}{.4}$ or $n = 150.0625$. Thus, $n = 151$ men should be chosen.

7.28 **a.** The population from which we are randomly sampling $n = 35$ measurements is not necessarily normally distributed. However, the sampling distribution of $\overline{X}$ does have an approximate normal distribution, with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$. The probability of interest is

$$P\left[|\overline{X}-\mu| < 1\right] = P\left[-1 < (\overline{X}-\mu) < 1\right].$$

Since

$$Z = \frac{\overline{X}-\mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

has a standard normal distribution, we need only find $\frac{\sigma}{\sqrt{n}}$ to approximate the above probability. Though σ is unknown, it can be approximated by $s = 12$ and $\frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{35}} = 2.028$. Then

$$P\left[|\overline{X}-\mu| < 1\right] = P\left[-\tfrac{1}{2.028} < Z < \tfrac{1}{2.028}\right] = P[-.49 < Z < .49] = 1 - 2(.3121)$$
$$= .3758.$$

b. No. The population mean μ is only the average of the estimates. It is possible that all of the estimates are too high, for example.

7.36 Let Y_i be the volume of the i^{th} sample. Then the total volume of the composite sample is ΣY_i, and this total must exceed 200 with probability .95. That is,

$$P(\Sigma Y_i > 200) = P\left(\overline{Y} - \mu > \tfrac{200}{50} - \mu\right) = P\left(Z > \frac{4-\mu}{\sqrt{\frac{4}{50}}}\right) = .95$$

Hence $(4 - \mu)\sqrt{\frac{4}{50}} = -1.645$ and $\mu = 4.47$.

7.46 Use the approximation given in Example 7.10. The quantity $Y_n = \frac{\frac{Y}{n}-p}{\sqrt{\frac{p(1-p)}{n}}}$ converges in distribution to a standard normal random variable. Hence, with $p = .10$ and $n = 100$, we have

$$P(Y \geq 15) \approx P\left(z \geq \tfrac{14.5-10}{\sqrt{9}}\right) = P(z \geq 1.5) = .0668$$

7.52 Refer to Exercise 7.51. If $p = .9$, then

$$P\left(\left|\tfrac{Y}{n} - p\right| \leq .15\right) = P\left(|Z| \leq \frac{.15}{\sqrt{\frac{(.9)(.1)}{50}}}\right) = P(|Z| \leq 3.54) = 1.00$$

7.56 As 80% of the disk contain no missing pulses, 20% contain missing pulses. Let X represent the count of our sample that have missing pulses. Then X is binomial with $p = .2$ and $n = 100$. We want

$$P(X \geq 15) = P(X \geq 14.5) \approx P\left(Z \geq \tfrac{14.5-(.2)100}{\sqrt{(.2)(.8)100}}\right) = P(Z \geq -1.38) = 1 - .0838 = .9162.$$