

Problem:

Refer to problem 7.3 of your text. How would you solve the problem if you did not know the population standard deviation and instead you only knew the sample standard deviation $s = 4$?

Answer:

$$\begin{aligned}
 P(|\bar{Y} - \mu| < 2) &= P(-2 < \bar{Y} - \mu < 2) \\
 &= P\left(\frac{-2}{\frac{4}{\sqrt{9}}} < \frac{\bar{Y} - \mu}{\frac{s}{\sqrt{n}}} < \frac{2}{\frac{4}{\sqrt{9}}}\right) \\
 &= P(-1.5 < T^8 < 1.5)
 \end{aligned}$$

So, you look this up on the t-table, but you don't find the number 1.5 in the row for 8 degrees of freedom. So you try to look at the values that bracket this one.

$$\begin{array}{ccccc}
 P(-1.397 < T^8 < 1.397) & < & P(-1.5 < T^8 < 1.5) & < & P(-1.860 < T^8 < 1.860) \\
 1 - 2(0.100) & < & P(-1.5 < T^8 < 1.5) & < & 1 - 2(0.050) \\
 0.80 & < & P(-1.5 < T^8 < 1.5) & < & 0.90 \\
 0.80 & < & P(|\bar{Y} - \mu| < 2) & < & 0.90
 \end{array}$$

(Using S-Plus, we can find out that this probability is more closely approximated by 0.8280.)