10.2 The test statistic Y has a binomial distribution with $n = 20$ and p.

a. A Type I error occurs if the experimenter concluded that the drug dosage level induces sleep in less than 80% of the people suffering from insomnia when, in fact, drug dosage level does induce sleep in 80% of insomniacs.

b. $\alpha = P(\text{reject } H_0 | H_0 \text{ true}) = P(Y \leq 12 | p = .8) = .032$, using Table 1, Appendix III.

c. A Type II error would occur if the experimenter concluded that the drug dosage level induces sleep in 80% of the people suffering from insomnia when, in fact, fewer than 80% experience relief.

d. If $p = .6$,

$$\beta = P(\text{accept } H_0 | H_0 \text{ false}) = P(Y > 12 | p = .6) = 1 - P(Y \leq 12p = .6) = 1 - .584$$
$$= .416$$

e. If $p = .4$, then

$$\beta = P(Y > 12 | p = .4) = 1 - P(Y \leq 12 | p = .4) = 1 - .979 = .021.$$

10.8 The parameter of interest is μ, the average daily wage of workers in a given company. The objective is to determine whether this company pays inferior wages in comparison to the total industry. Thus the hypothesis to be tested is

$$H_0:\ \mu = 13.20 \qquad \text{vs.} \qquad H_a:\ \mu < 13.20$$

The best estimator for μ is the sample average, $\overline{y} = 12.20$. The test statistic is

$$Z = \frac{\overline{Y} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

which represents the distance (measured in units of standard deviation) from $\overline{Y}$ to the hypothesized mean μ. Calculating the value of the test statistic using the

information contained in the sample, we have

$$z = \frac{\overline{y} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{12.20 - 13.20}{\frac{2.50}{\sqrt{40}}} = \frac{-\sqrt{40}}{2.5} = -2.53$$

The critical value of Z that separates the rejection and non rejection regions will be a value (denoted by z_0) such that $P(Z < z_0) = .01$. That is, $z_0 = -2.326$ (see Figure 10.2). The null hypothesis will be rejected if $z < -2.326$. Note that the observed value of the test statistic falls in the rejection region. Thus the conclusion is to reject the null hypothesis. There is evidence to indicate that this company is paying inferior wages.

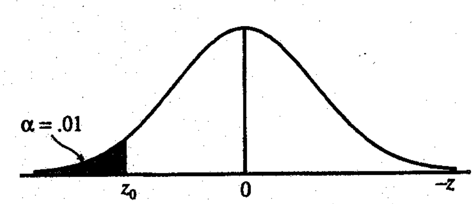

Figure 10.2

10.11 We are to test $H_0:\ \mu_1 - \mu_2 = 0$ vs. $H_a:\ \mu_2 - \mu_2 \neq 0$. The test statistic and rejection region are

$$z = \frac{1.65 - 1.43}{\sqrt{\frac{(.26)^2}{30} + \frac{(.22)^2}{35}}} = 3.65$$

RR: Reject H_0 if $|z| > 2.575$.

Conclusion: Reject H_0 at $\alpha = .01$. The soils do appear to differ with respect to average shear strength, at the 1% significance level.

10.18 Throughout let p_1 be the relevant proportion in 1986 and p_2 be the appropriate proportion in 1991.

a. The hypothesis of interest is

$$H_0\colon\ p_1 - p_2 = 0 \qquad \text{vs.} \qquad H_a\colon\ p_1 - p_2 \neq 0$$

10.20 The manufacturer claims that at least 20% of the public prefer her product. In order to test this claim, the following hypothesis is employed:

$$H_0\colon\ p = .2 \qquad \text{vs.} \qquad H_a\colon\ p < .2$$

Rejection of the null hypothesis would imply that the acceptance and rejection regions will be $z = -1.645$, since values of $\widehat{p}$ in the lower tail of the distribution will tend to disprove the null hypothesis (see Figure 10.3). The objective, then, is to determine a value for $\widehat{p}$ such that the corresponding test statistic z will be less than or equal to -1.645. Under the assumption of the null hypothesis, $p = .2$ and

$$\sigma_{\widehat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{(.2)(.8)}{100}} = .04$$

A value for $\widehat{p}$ must be found so that

$$z = \frac{\widehat{p}-.2}{.04} \leq -1.645$$

Solving for $\widehat{p}$ yields $\widehat{p} \leq .1342$.

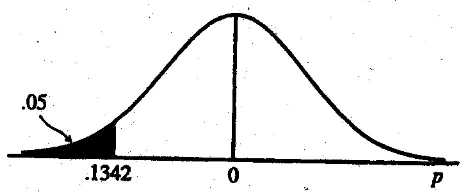

Figure 10.3

Calculate

$$\widehat{p}_1 = .45,\ \widehat{p}_2 = .34, \text{ and } \widehat{p} = \frac{n_1\widehat{p}_1 + n_2\widehat{p}_2}{n_1+n_2} = \frac{450+340}{1000+1000} = .395$$

The test statistic is then

$$z = \frac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}\widehat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{.45-.34}{\sqrt{(.395)(.605)\left(\frac{1}{1000}+\frac{1}{1000}\right)}} = 5.03.$$

The rejection region with $\alpha = .05$ is $z > 1.96$ and H_0 is rejected. There is evidence of a difference in the proportion of users in 1986 and 1991.

b. The hypothesis of interest is

$$H_0\colon\ p_1 - p_2 = 0 \qquad \text{vs.} \qquad H_a\colon\ p_1 - p_2 < 0$$

Calculate

$$\widehat{p} = .14,\ \widehat{p}_2 = .26, \text{ and } \widehat{p} = \frac{n_1\widehat{p}_1 + n_2\widehat{p}_2}{n_1+n_2} = \frac{140+260}{1000+1000} = .2$$

The test statistic is then

$$z = \frac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}\widehat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{.14-.26}{\sqrt{(.2)(.8)\left(\frac{1}{1000}+\frac{1}{1000}\right)}} = -6.71.$$

The rejection region with $\alpha = .05$ is $|z| < -1.645$ and H_0 is rejected. There is evidence to indicate that ibuprofen has significantly increased its market share from 1986 to 1991.

c. Yes. The survey is based on the same samples of 1000 people. Hence, if a person has decreased his use of aspirin, he may have begun using ibuprofen. The tests are not independent.

10.29 In Exercise 10.20 we found that we reject when $\widehat{p} \leq .1342$. Thus

$$\beta = P(\widehat{p} > .1342).$$

The corresponding z value is

$$z = \frac{.1342-.15}{\sqrt{\frac{(.15)(.85)}{100}}} = -.44$$

Hence

$$\beta = P(Z > -.44) = P(Z < .44) = 1 - .3300 = .6700.$$

10.36 The rejection region is

$$\frac{\widehat{\theta}-\theta_0}{\sigma_{\widehat{\theta}}} > z_\alpha$$

which occurs if and only if

$$\widehat{\theta} - \theta_0 > z_\alpha \sigma_{\widehat{\theta}}$$

which occurs if and only if

$$\widehat{\theta} - z_\alpha \sigma_{\widehat{\theta}} > \theta_0$$

where the left-hand side is the $100(1-\alpha)\%$ lower confidence bound for θ.

10.40 We are to test

$$H_0\colon \mu \geq .6 \qquad \text{vs.} \qquad H_a\colon \mu < .6.$$

The test statistic is

$$z = \frac{\overline{y}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{.58-.6}{\frac{.11}{\sqrt{120}}} = -1.99$$

The p-value is

$$p\text{-value} = P(Z < -1.99) = .0233$$

Since $.0233 < .10$, we would reject H_0 in a test at level $\alpha = .10$.

10.44 a. The hypothesis to be tested is

$$H_0\colon p = .85 \qquad \text{vs.} \qquad H_a\colon p > .85$$

where p is the proportion of right-handed executives of large corporations. The test statistic is

$$z = \frac{\widehat{p}-p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.96-.85}{\sqrt{\frac{(.85)(.15)}{300}}} = 5.34.$$

The rejection region, with $\alpha = .01$, is $z > 2.33$ and H_0 is rejected. The percentage of right-handed executives is greater than the proportion of right-handed people in the general population.

b. For a one-tailed test,

$$p\text{-value} = P[z > 5.34] < .001.$$

Hence, H_0 can be rejected for any value of $\alpha > .001$.

10.46 We are to test

$$H_0\colon \mu_1 - \mu_2 = 0 \qquad \text{vs.} \qquad H_a\colon \mu_1 - \mu_2 > 0.$$

The test statistic is

$$z = \frac{6.9-5.8}{\sqrt{\frac{(2.9)^2}{35}+\frac{(1.2)^2}{35}}} = 2.074$$

The p-value (attained significance level) is

$$p\text{-value} = P(Z > 2.074) = .0192$$

The company would reject H_0 at the level $\alpha = .05$.

10.54 The hypothesis to be tested is

$$H_0\colon \mu = 100 \qquad \text{vs.} \qquad H_a\colon \mu < 100$$

Calculate

$$\overline{y} = \frac{\sum y_i}{n} = \frac{1797.095}{20} = 89.85475$$

$$s^2 = \frac{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}{n-1} = \frac{165{,}697.7081 - \frac{(1797.095)^2}{20}}{19} = 222.115067$$

The test statistic is

$$t = \frac{\overline{y}-\mu}{\frac{s}{\sqrt{n}}} = \frac{89.85475-100}{\sqrt{\frac{222.115067}{20}}} = -3.05$$

The critical value of t with $\alpha = .01$ and $n-1 = 19$ degrees of freedom is $t_{.01} = 2.539$, and the rejection region is $t < -2.539$. The null hypothesis is rejected and we conclude

10.58 The hypothesis to be tested is

$$H_0\colon \mu_1 - \mu_2 = 0 \qquad \text{vs.} \qquad H_a\colon \mu_1 - \mu_2 > 0.$$

Calculate

$$s^2 = \frac{9(.017)^2+12(.006)^2}{21} = .00014443$$

The test statistic is then

$$t = \frac{\overline{y}_1-\overline{y}_2}{\sqrt{s^2\left[\left(\frac{1}{n_1}\right)+\left(\frac{1}{n_2}\right)\right]}} = \frac{.041-.026}{\sqrt{s^2\left[\left(\frac{1}{10}\right)+\left(\frac{1}{13}\right)\right]}} = 2.97$$

The rejection region, with $\alpha = .05$ and 21 degrees of freedom, is $t > 1.721$, and the null hypothesis is rejected.