Review of Set Theory

1 Sets

A **set** is a well-defined collection of objects or elements. Sets are denoted by capital letters. If an element a belongs to a set A, it is expressed by the notation $a \in A$. If a does not belong to A, it is expressed by the notation $a \notin A$.
Example: Set of all students in a class, set of positive integers, set of possible numbers you may get if you roll a die.

A set can be written in two ways:

- by listing all elements in the set.
  Example: The set of possible outcomes in a roll of a die $= \{1, 2, 3, 4, 5, 6\}$

- by writing the properties of the elements of the set.
  Example: The set of possible outcomes in a roll of a die $= \{x \mid x$ is an integer and $1 \leq x \leq 6\}$

Subset: A subset is a part of a set. A set B is a subset of a set A if each element of B is also an element of A. If B is a subset of A, that is denoted by $B \subset A$.
Example: $\{1, 2, 4\}$ is a subset of $\{1, 2, 3, 4, 5\}$, i.e., $\{1, 2, 4\} \subset \{1, 2, 3, 4, 5\}$
Superset: A set B is a superset of a set A if A is a subset of B. If B is a superset of A, that is denoted by $B \supset A$.
The Null Set: The null set is the set with no elements. It is denoted by $\{\}$ or Φ.
Universal set: Often, in a particular context, all the sets are considered to be subsets of a bigger set U, which is called the universal set. In Probability theory, the sample space S (the set of all possible outcomes) is taken as the universal set.

Venn diagram is a graphical representation of sets. Usually the universal set is represented by a rectangle and other sets are represented by parts of the rectangle.

Union of two sets A and B is the set of elements which are either in A or in B or in both. Union of A and B is denoted by $A \cup B$. Similarly, we can define union of three or more sets. *Note that $A \cup B$ means A **OR** B.*
Example: $A = \{1, 2, 6\}, B = \{1, 3, 4, 6\}$. Then $A \cup B = \{1, 2, 6, 3, 4\}$.
Note that the elements are not repeated in a set.

Intersection of two sets A and B is the set of elements which are in both A and B. Intersection of A and B is denoted by $A \cap B$. Similarly, we can define intersection of three or more sets. *Note that $A \cap B$ means A **AND** B.*
Example: Consider A and B as in the above example. Then $A \cap B = \{1, 6\}$.

Two sets A and B are said to be **disjoint** if they do not have any elements in common, i.e., if $A \cap B = \Phi$. This definition can be extended for more than two sets.
Example: $A =$ the set of clubs in a standard deck of cards, $B =$ the set of spades in a standard deck of cards. Then $A \cap B = \Phi$. So, A and B are disjoint sets.

The **complement** of a set A is defined to be the set elements which are not in A and is denoted by A^c. It is precisely the set of elements in U (the universal set) which are not in A. *A^c means* **NOT** *A*

Example: U = The set of numbers on a die = $\{1, 2, 3, 4, 5, 6\}$, $A = \{1, 4, 6\}$. Then $A^c = \{2, 3, 5\}$.

2 Some Laws of Sets

In this section we assume that A, B, C are three sets.

Commutative laws:

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

Associative laws:

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive laws:

- $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
- $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

DeMorgan's laws:

- $(A \cup B)^c = A^c \cap B^c$
- $(A \cap B)^c = A^c \cup B^c$