

Solutions for quiz 4

1. Let X be the number of games that the Cubs wins. Then X follows a binomial distribution with $n=5$ and $p=.4$.

(a)

$$\begin{aligned}P(X \geq 3) &= \sum_{x=3}^5 \binom{5}{x} (.4)^x (.6)^{5-x} \\&= \binom{5}{3} (.4)^3 (.6)^2 + \binom{5}{4} (.4)^4 (.6) + \binom{5}{5} (.4)^5 \\&= .2304 + .0768 + .01024 = .31744\end{aligned}$$

(b)

$$P(X \geq 1) = 1 - P(X = 0) = 1 - (.6)^5 = .92224$$

2. Let X be the test score.

(a)

$$\begin{aligned}P(400 \leq X \leq 480) &= P\left(\frac{400 - 420}{80} \leq \frac{X - 420}{80} \leq \frac{480 - 420}{80}\right) \\&= P(-.25 \leq Z \leq .75) \\&= 1 - P(Z \leq -.25) - P(Z \geq .75) \\&= 1 - .4013 - .2266 = .3721\end{aligned}$$

- (b) Let x_0 be the minimum score needed to be in the top 20%. Then it is necessary to have

$$\begin{aligned}P(X \geq x_0) &= .2 \\P(Z \geq \frac{x_0 - 420}{80}) &= .2 \\ \frac{x_0 - 420}{80} &= .84 \\x_0 = 420 + (.84)(80) &= 487.2\end{aligned}$$