

Solutions for quiz 5

1. (a)

$$\begin{aligned}
 c \int_0^1 \int_{-3}^{-1} (x - y) dy dx &= c \int_0^1 \left(xy - \frac{y^2}{2} \right) \Big|_{-3}^{-1} dx \\
 &= c \int_0^1 \left(-x - \frac{1}{2} + 3x + \frac{9}{2} \right) dx \\
 &= c \int_0^1 (2x + 4) dx = c(x^2 + 4x) \Big|_0^1 = 5c = 1 \\
 c &= \frac{1}{5}
 \end{aligned}$$

(b)

$$\begin{aligned}
 P(X > .5, Y \leq -2) &= \frac{1}{5} \int_{.5}^1 \int_{-3}^{-2} (x - y) dy dx \\
 &= \frac{1}{5} \int_{.5}^1 \left(xy - \frac{y^2}{2} \right) \Big|_{-3}^{-2} dx = \frac{1}{5} \int_{.5}^1 (-2x - 2 + 3x + \frac{9}{2}) dx \\
 &= \frac{1}{5} \int_{.5}^1 \left(x + \frac{5}{2} \right) dx = \frac{1}{5} \left(\frac{x^2}{2} + \frac{5}{2}x \right) \Big|_{.5}^1 \\
 &= \frac{1}{5} \left(\frac{1}{2} + \frac{5}{2} - \frac{1}{8} - \frac{5}{4} \right) = \frac{13}{40}
 \end{aligned}$$

(c)

$$\begin{aligned}
 f(x, y) &= \frac{1}{5}(x - y) \\
 f(x) &= \frac{1}{5} \int_{-3}^{-1} (x - y) dy = \frac{1}{5} \left(xy - \frac{y^2}{2} \right) \Big|_{-3}^{-1} = \frac{1}{5}(2x + 4) \\
 f(y) &= \frac{1}{5} \int_0^1 (x - y) dx = \frac{1}{5} \left(\frac{x^2}{2} - xy \right) \Big|_0^1 = \frac{1}{5} \left(\frac{1}{2} - y \right) \\
 f(x)f(y) &= \frac{1}{25}(2x + 4) \left(\frac{1}{2} - y \right) \neq f(x, y)
 \end{aligned}$$

X and Y are not independent.