

Solutions for quiz 6

1. (a)

$$V(X) = E(X^2) - (E(X))^2 = 6 - (2)^2 = 6 - 4 = 2$$

(b) X and Y are independent. Therefore,

$$E(XY) = E(X)E(Y) = 2 \cdot 1 = 2$$

(c) X and Y are independent and hence $\text{Cov}(X, Y) = 0$. Therefore,

$$V(2X + 3Y) = V(2X) + V(3Y) = 4V(X) + 9V(Y)$$

$$V(Y) = E(Y^2) - (E(Y))^2 = 3 - 1 = 2$$

$$V(2X + 3Y) = 4 \cdot 2 + 9 \cdot 2 = 26$$

(d)

$$\text{Cov}(2X + 3Y, X) = 2\text{Cov}(X, X) + 3\text{Cov}(Y, X)$$

$$= 2V(X) = 4$$

(e)

$$\rho(2X + 3Y, X) = \frac{\text{Cov}(2X + 3Y, X)}{\sqrt{V(2X + 3Y)}\sqrt{V(X)}}$$

$$= \frac{4}{\sqrt{26}\sqrt{2}} \simeq .555$$