

Solutions for quiz 7

1. (a) Using the Central Limit Theorem, $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ will converge to standard normal distribution as the sample size gets large. Here $n = 225$ and $\sigma = 9500$, therefore,

$$\begin{aligned}P(\bar{X} < 56200) &= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{56200 - \mu}{\sigma/\sqrt{n}}\right) \\&= P\left(Z < \frac{56200 - 55000}{9500/\sqrt{225}}\right) = P(Z < 1.894) \\&= 1 - .0294 = .9706\end{aligned}$$

- (b) No. We do not need to assume the population distribution to be normal to use the Central Limit Theorem.

2. (a) The T-statistic $\frac{\bar{X}-\mu}{S/\sqrt{n}}$ will have a t distribution with $n - 1$ degrees of freedom if we are sampling from a normal population. Here $n = 16$ and $S = 3$,

$$\begin{aligned}P(\bar{X} - \mu > 1.5) &= P\left(\frac{\bar{X} - \mu}{S/\sqrt{n}} > \frac{1.5}{S/\sqrt{n}}\right) \\&= P\left(T > \frac{1.5}{3/\sqrt{16}}\right) = P(T > 2)\end{aligned}$$

Looking at the t table, the T-value for a .025 upper-tail with $n-1 = 15$ degrees of freedom is 2.131, therefore, $P(T > 2) \simeq .025$.

- (b) No.