Solutions for quiz 8

1. The confidence interval for the difference between proportions of failures has the form

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Here $\hat{p}_1 = .1$, $\hat{p}_2 = .03$, $n_1 = n_2 = 100$,and $Z_{.02} = 2.05$. The 96% confidence interval is

$$(.1 - .03) \pm 2.05\sqrt{\frac{(.1)(.9)}{100} + \frac{(.03)(.97)}{100}}$$

$$= .07 \pm 2.05 * .0345$$

$$= [0, .1407]$$

2. (a)

$$\mathrm{E}(\hat{\theta}) = \frac{\theta + (\theta + 1)}{2} = \theta + \frac{1}{2} \neq \theta$$

No, $\hat{\theta}$ is not unbiased.

(b)

$$\mathrm{Var}(\hat{\theta}) = \frac{1}{n}\frac{(\theta + 1 - \theta)^2}{12} = \frac{1}{12n}$$

(c)

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}(\hat{\theta}) + (\mathrm{Bias})^2$$

$$= \frac{1}{12n} + (\frac{1}{2})^2 = \frac{3n+1}{12n}$$