STA244 **2/14/2005**

Homework 4

Due 2/21/2001

1. Consider the linear model

$$Y = X_1\beta_1 + X_2\beta_2 + \epsilon$$

where X_1 is $n \times q$ and X_2 is $n \times (p-q)$, with both matrices of full column rank. Consider the problem of testing $NH : \beta_1 = 0$. Assume that $\epsilon \sim N(0, \sigma^2 I_n)$.

   (a) Give a basis for Ω (the parameter space for $\mu = E(Y)$)?

   (b) Give a basis for Ω_o (the parameter space for μ under the NH)?

   (c) Find the MLE's of μ and σ^2 under the NH.

   (d) Show that $Z = Q_{X_2}X_1$ is a basis for $\Omega - \Omega_o$ and that the span of $\{Z, X_1\} = \Omega$. Also show that $P_Z + P_{X_2} = P_\Omega$ where P_Z and P_{X_2} are orthogonal projections and that $P_Z P_{X_2} = 0$.

   (e) Construct an ANOVA table for testing the NH, filling in entries with the appropriate df, quadratic forms using Q_Ω and Q_{Ω_o}, etc. This is sometimes called the Extra Sum of Squares F-test.

   (f) Use the above result to test the hypotheses in Exercise 11.2 in CW. (do only the first 4 tests)

2. Consider testing $NH : B(\beta - a) = 0$ in the linear model where

$$Y \sim N(X\beta, \sigma^2 I_n)$$

and B is an $r \times p$ matrix, $r \leq p$ and β and a are $p \times 1$ vectors. Derive an F-test for testing the NH by finding a 1-to-1 reparameterization of the model, so that you can use your previous result. Linear combinations that involve no unknown parameters are an *offset*, and can be used to define a new response variable.

3. Apply the above test in exercise 11.1 in CW. In S-Plus, one can handle offsets directly in the linear model function:

```
lm.obj <- lm(log(Vol) - 2*log(D) ~ log(D) + log(Ht), data=bcherry)
```

The functions `summary(lm.obj)` and `anova(lm.obj)` can be used to obtain various summaries of the model. An alternative way to fit the model with an offset is to use the `glm()` function:

```
lm.obj <- glm(log(Vol)~ log(D) + log(Ht) + offset(2*log(D)), data=bcherry)
```

Use `summary(lm.obj)` to get SE's etc.

4. Exercise 9.2 in CW (see class handout for fitting AOV's in S-Plus)

5. Exercise 9.3 in CW