- **Time and Places of Exam:** The exam will be in class, on Thursday, Feb 13 2003.

- **Exam Materials:** The exam is closed book. A few pages of formulas and tables will be appended to the exam.

  You can also prepare your own formula sheet: a single one-sided 8x11 sheet on which you can write whatever you like. You should bring a calculator.

- **Exam Coverage:** Questions for the exam will be based on material in the following sections of your textbook:

  Chapter 2 : Entire chapter.

  Chapter 3 : Sections 2-8.

  Chapter 4 : Sections 1-10.

  Chapter 5 : Sections 1-9.

Descriptive Statistics, Chap 2

- Types of data: qualitative and quantitative
- population and sample, parameter and statistics
- mean, variance and standard deviation of a sample
- median, lower (upper) quartile and 100pth percentile
- z-score, IQR and outliers
- Boxplot, histogram and bar plot

Boxplot

- Calculate the median m, lower and upper quartile, Q_l and Q_u, and the interquartile range, IQR.

- draw a box with ends at Q_l and Q_u and draw a vertical line inside the box to locate the median m.

- Draw whiskers out to $Q_u + 1.5\text{IQR}$ and $Q_l - 1.5\text{IQR}$, **but** trim them back to the most extreme data point in those ranges.

- Draw dots for each individual data point outside the box and whiskers.

Example: Ex 2.54

Probability, Chap 3

- Sample space, events, Compound events (union, intersection and complement)

- Probability, conditional probability and probability rule for compound events

- Independence and mutually exclusive

- Bayes theorem

- Counting rules

Example: In the week following final exams, 10% of Duke students go to the beach (it always seems like more); 50% go home; and 40% start a job. Of the beach-goers, 90% (claim to) have fun; 22% of the home-goers have fun; and a disappointing 25% of the workers have fun.

- a. What is the probability that a randomly-selected Duke student will go to the beach and not have fun?
- b. What is the probability that a randomly-selected Duke student will have fun (anywhere)?
- c. What is the probability that a randomly-selected Duke student who **IS** having fun can see the ocean?
- d. Don't you wish you had a job working at home and that you lived at the beach?

Example: A communications satellite is launched into orbit on an three-stage rocket. The first stage of the rocket fails with probability 0.10 and the second and third stage are more reliable, each failing with probability 0.01. The satellite itself has a rocket, which places it into geosynchronous orbit, failing with probability 0.001. Assuming that all stages of the rocket, and the satellite itself, fail (and succeed) independently:

- a. What is the probability that the satellite is successfully placed into the desired orbit? (0.8812)

- b. Given that the first stage works successfully, what is this success probability? (0.9791)

- c. Given that the first stage does not fail, what is the probability that the second stage does not fail? (0.99)

- d. You are told that the satellite did not make it into orbit, but nothing more. What is the probability that the first stage failed? (0.8418)

Counting Rule

- Basic rule: Multiplicative rule

- Important setting:
  - With replacement or Without
  - Order matters or not

- Other rules:
  - Permutations $n!$
  - Combinations (choose k out of n) $\frac{n!}{k!(n-k)!}$.
  - Partitions $\frac{n!}{n_1! \cdots n_k!}$

Ex 3.44: The Federal Aviation Administration (FAA) has a 16-member committee working on evaluating the traffic control systems of four facilities replying on computer-based equipment. How many possible ways the FAA can form the task force to do the evaluation,

- a. if the FAA wants to assign one member to each facility and assume that each member can be assigned to more than one facilities?

- b. if the FAA wants to assign one member to each facility and assume that each member can NOT be assigned to more than one facilities?

- c. if the FAA wants to form a 4-memeber task force for all the four facilities?

- d. if the FAA wants to assign 4-member task force to each facilities?

a. 16^4

b. $P_4^{16} = (16)(15)(14)(13)$

c. $\binom{16}{4} = \frac{(16)(15)(14)(13)}{(4)(3)(2)(1)}$

d. $\frac{16!}{4!4!4!4!}$

Random Variables, Chap 4-5

- Discrete or continuous
- Probability distribution: probability mass function (discrete), probability density (continuous)
- Cumulative distribution function and its use in calculating probabilities for cntinuous random variables
- Expectation of functions of random variables: mean, variance (standard deviation)

Important Distributions

Discrete

- Bernoulli(p), $x = 0, 1$
- Binomial(n, p), $x = 0, 1, \ldots, n$
- Geometric(p), $x = 0, 1, \ldots, n, \ldots$
- Poisson(λ), $x = 0, 1, \ldots, n, \ldots$
- Discrete uniform

Continuous

- Uniform(a, b), $x \in [a, b]$
- Normal(μ, σ^2), $x \in \mathbb{R}$
- Exponential(β), $x \geq 0$

Distributions and Stories

Semiconductor Wafter : A semiconductor wafer contains a contamination particle with probability $p = 0.01$.

- Whether a new produced wafter is contaminated or not – Bernoulli(p)
- How many are contaminated among 1000 wafers? – Binomial(n, p)
- How many wafters will we look at before we find a contaminated one? – Geometric(p)
- How many wafters will we look at before we find the kth contaminated wafer? – Negative Binomial
- Suppose today's production is 1000 wafers and among them 10 are contaminated. Now randomly draw a sample of 100 wafers from today's production, how many are contaminated? – Hypergeometric

Fishing Example : You go to a spot in Duke Forest to try to catch some fish. From past experience you know that the average number of fish you catch per hour is $\lambda = 6$.

- How many fish you catch in one hour of fishing? – Poisson(λ)

- How many fish you catch in 20 minutes? – Poisson($\lambda/3$)

- How long you have to wait to catch your first fish? – Exponential(β) with $\beta = 1/\lambda$.

- How long you have to wait to catch your αth fish? – Gamma(α, β) with $\beta = 1/\lambda$.

Normal Distribution

- Two parameter: mean μ and variance σ^2 (standard deviation σ)

- If $x \sim N(\mu, \sigma^2)$ then $(x - \mu)/\sigma$ is distributed as the standard normal $N(0, 1)$

- Calculation of normal probabilities using the standard normal random variable and the table in Appendix

Example: The number of cracks in a section of $I-85$ which need to be repaired is assumed to be a Poisson random variable with parameter $\lambda = 1$ crack per mile.

- a. What's the probability that no cracks require repair in 5 miles of highway? (0.0067)
- b. What's the probability that more than one crack requires repair in 5 miles of highway? (0.9596)
- c. The highway is inspected and repaired in one mile section. What's the probability that no cracks are found until mile 5? (0.0116)
- d. What's the probability that 1 of the first 5 sections inspected has cracks, but the other 4 do not? (0.0579)

Example: The life of a semiconductor laser is normally distributed with a mean of 7000 hours and a standard deviation of 600 hours

- a. What's the probability that a laser lasts less than 6000 hours? (0.0475)

- b. What's the probability that a laser lasts more than 8000 hours? (0.0475)

- c. What's the lifetime in hours exceeded by 99% of all lasers? (5602)

- d. Seven lasers are required in an industrial system. What's the probability that all seven function for at least 6000 hours? ($(1 - 0.0475)^7$)