Homework 2

Due 1/22/2003

1. Write the following two way analysis of variance (AOV) model with interactions

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$

with $i = 1, 2, 3$, $j = 1, 2$, $k = 1, 2$ in matrix notation.

2. Suppose we have a $k \times k$ matrix S partitioned as

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where S_{11} is $p \times p$, S_{22} is $q \times q$, $S_{12} = S_{21}'$ and $k = p + q$.

(a) If S is non-singular, show that

$$S^{-1} = \begin{bmatrix} S_{11}^{-1} + BS_{22.1}^{-1}B' & -BS_{22.1}^{-1} \\ -S_{22.1}^{-1}B' & S_{22.1}^{-1} \end{bmatrix}$$

where $S_{22.1} = S_{22} - S_{21}S_{11}^{-1}S_{12}$ and $B = S_{11}^{-1}S_{12}$.

(b) Let W denote the $n \times k$ partitioned matrix

$$W = [\mathbf{1}_n | X]$$

with $\mathbf{1}_n$ denoting the $n \times 1$ column ones and X denoting the remaining $n \times p$ matrix of regressors. Find the partitions (blocks) of $W'W$ in terms of n, the vector of sample means $\bar{x}$, and $X'X$.

(c) Find $(W'W)^{-1}$ using the result from (a). Simplify the expression in terms of the $p \times p$ corrected sum of squares matrix $(X - 1_n\bar{x}^t)'(X - 1_n\bar{x}^t))$.

(d) If $\mathcal{W} = [\mathbf{1}_n | \mathcal{X}]$ with $\mathcal{X} = (X - 1_n\bar{x}^t)$, the matrix of centered regressors, find $(\mathcal{W}'\mathcal{W})^{-1}$.