Overview of Unit #1: Point Estimators

There were 5 subunits in this unit, each devoted to a particular class of estimators.

1. Bayes estimators: Refs: BC7.2.3, BD3.2

Background: loss, risk, posterior risk, minimax.

Major results: Derived posterior mean, median under squared error loss and absolute loss.

Comment: Taken together, this is existence and uniqueness for Bayes point estimators.

2. Best Unbiased/UMVUE's: Mostly from BC7.3.2, 7.3.3 (not BD) supplemented from other sources (Lehmann's books which are not required but the results are important here.)

Background: MSE, variance/bias decomposition, Jensen's inequality (for strictly convex functions), 2 extra lectures on sufficiency and completeness.

Major results:

A) delta UMVU iff uncorrelated with estimators of 0.
B) Rao-Blackwell: conditioning on sufficient statistics only improves risk.
C) Lehmann-Scheffe Theorem: T complete, sufficient for theta, then every unbiasedly estimable function of theta has a unique unbiased estimator.
D) Examples to see how UMVU extends to nonparametric cases.

Comment: Taken together this is existence and uniqueness for UMVUE's.

3. Method of Moments: BC7.2.1, BD2.1.1

Background: contrast functions, estimating equations

Defined the procedure in general.

4. MLE's BC:7.2.2 BD: 2.2.2, 2.2.3, 2.3 -- Mostly from BD, not BC

Major Results:

A) invariance of the MLE
B) existence of MLE, behavior at boundary
C) uniqueness
D) behavior of MLE in exponential families, existence, uniqueness, form of likelihood equation (multiparameter case too)

Taken together, this is existence and uniqueness for MLE's.

5. Fisher information BD3.4.2, BC 7.3.2

Maor Results

A) Definition by first derivatives, second derivative form.
B) interpretation as relative flatness.
C) Cramer-Rao Lower Bound, necessity of derivative condition
D) Define extension to multiparameter case, invariance

I omitted: equivariant estimators, EM and other algorithms.