STA226 Homework #1
Spring 2004 Jan 30, 2004

Homework 1

Due: Friday, Feb 13, 2004

1. An insurance company provides insurance policies against hurricane damages. you are specially concerned about tree damage (E) and flood damage (F). In order to state the policies you are asked to give your rates for the following events: $E, F, E \mid F$ and $F \mid E$.
Suppose your rates are as follows: 0.2 for E; 0.3 for F, 0.8 for $E \mid F$ and 0.9 for $F \mid E$.
Show that the broker can establish stakes such that you are a sure loser.

2. Show that for every $\delta \in D^*$,

$$\sup_{\pi \in \Theta^*} r(\pi, \delta) = \sup_{\theta \in \Theta} R(\theta, \delta).$$

3. Let $\Theta = (0, \infty)$, let $\mathcal{A}$ be the real line, and let $L(\theta, a) = (\theta - a)^2$. Let the distribution of X be Poisson with parameter $\theta > 0$,

$$p(x \mid \theta) = e^{-\theta} \frac{\theta^x}{x!} \quad x = 0, 1, \ldots.$$

Take the prior distribution of θ as the gamma distribution $\mathcal{G}(\alpha, \beta)$ with density

$$\pi(\theta) = \frac{\theta^{\alpha-1}}{\Gamma(\alpha)\beta^\alpha} e^{-\theta/\beta} \quad \theta > 0,$$

where $\alpha, \beta > 0$.
(a) Show that the Bayes rule is $d(x) = \beta(\alpha + x)/(\beta + 1)$.
(b) Show that the usual estimator (mle) $d(x) = x$ is not a Bayes rule.
(c) Show that $d(x) = x$ is a generalized Bayes with respect to the improper prior $\pi(\theta) = 1/\theta$.

4. Let $\Theta = (0, 1)$, $\mathcal{A} = [0, 1]$ and $L(\theta, a) = (\theta - a)^2$. Let the distribution of X be binomial with n trials and probability θ of success.
(a) Find the Bayes rule with respect to the beta prior

$$\pi(\theta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1}(1 - \theta)^{\beta-1}, \quad 0 < \theta < 1$$

where $\alpha, \beta > 0$.
(b) If the loss is changed to be $L(\theta, a) = (\theta - a)^2/[\theta(1-\theta)]$. Show that the usual mle $d(x) = x/n$ is Bayes with respect to the uniform prior on $(0, 1)$.