Homework 2

Due: Friday, Feb 27, 2004

1. Let $\Theta = (0, \infty)$, $\mathcal{A} = [0, \infty)$, $X \sim \text{Poisson}(\theta)$ and let $L(\theta, a) = (\theta - a)^2/\theta$.
(a) Show that $\delta_0(x) = x$ is an equalizer rule.
(b) Show that δ_0 is generalized Bayes with respect to $\pi(\theta) = 1$ on Θ.
(c) Show that δ_0 is minimax.

2. Suppose $\{f(x \mid \theta) : \theta \in \Theta\}$ has *monotone likelihood ratio*, that is, the likelihood ratio

$$\frac{f(x \mid \theta_2)}{f(x \mid \theta_1)}$$

is a non-decreasing function of x when $\theta_1 < \theta_2$. Show that for any fixed number c,

$$P[X > c \mid \theta_2] \geq P[X > c \mid \theta_1].$$

3. Assume $X \sim \text{Bino}(15, \theta)$ and it is desired to test $H_0 : \theta = 1/3$ versus $H_1 : \theta = 2/3$. Suppose $0 - K_i$ loss is used here with $K_0 = 2$ and $K_1 = 1$. Find the minimax test. [Take a look of example 18 on page 357 of JB.]

4. Ex 45 on page 293 of JB