STA226 Homework #3
Spring 2004 Feb 27, 2004

Homework 3

Due: Monday, Mar 15, 2004

1. Consider Gaussian random variable $X \sim N(\mu, I_m)$. Suppose we wish to test simultaneously the m null hypotheses $H_i : \mu_i = 0$ against the two-sided alternatives $H_i' : \mu_i \neq 0$. A simple test procedure would be to reject H_i if $|X_i| \geq z_{\alpha/2}$ where $z_{\alpha/2}$ is such that $\Phi(z_{\alpha/2}) = 1 - \alpha/2$ and $\Phi(\cdot)$ is the standard normal CDF. Assume that $\mu_1 = \cdots = \mu_{m_0} = 0$ and $\mu_{m_0+1} = \cdots = \mu_m = d$ where d is just a fixed nonzero number.

Derive analytical formulae for the following Type I error rates:
(a) PCER $= \mathbb{E}(V)/m$.
(b) FWER $= P(V \geq 1)$.
(c) $FDR = \mathbb{E}[V/R]$ with the convention that $0/0 = 0$. [you might find the following notation is useful: $\beta = 1 - \Phi(z_{\alpha/2} - d) + \Phi(-z_{\alpha/2} - d)$.]

2. Choose your values for m, m_0 and d in Ex(1), for example, $m = 10, m_0 = 5, d = 2$. Also choose a significant level α. Generate a m-vector X from $N(\mu, I_m)$. Apply *Benjamini and Hochberg procedure* to the multiple testing. Repeat it n times and report the Monte Carlo estimate for FWER, FDR and FNR $= \mathbb{E}[T/W]$.

Do the same simulation for *Bonferroni procedure.*

3. Consider two bags, H and K, with two balls each. Each ball is either black or white. A white ball is added to bag H and a hidden ball is transferred at random from bag H to bag K.
(a) What is the chance of drawing a white ball from bag K?
(b) Then, a hidden ball is transferred from bag K to bag H. What is the chance now of drawing a white ball from bag H?

4. Consider $X \sim N(\theta, 1)$ with $|\theta| \leq m$. Show that, for the quadratic loss, $\delta^m(x) = m \tanh(mx)$ is a Bayes estimator associated with the two-point prior putting each mass on $\pm m$.