STA226 Spring 2004

Homework #4 Apr 2, 2004

Homework 4

Due: Monday, Apr 19, 2004

1. Suppose the loss function is additive, that is, $L(a, \theta) = \sum_i L(a_i, \theta_i)$. Also suppose the likelihood factorizes over i, i.e., $p(y \mid \theta) = \prod_i p(y_i \mid \theta_i)$. Then, show that the risk for a Bayes estimator with respect to a product prior $\pi(\theta) = \prod_i \pi_i(\theta_i)$ is additive in θ_i.

2. Suppose that $Y \sim N(\theta, \epsilon^2 I)$ where $\theta \in \Theta$ and Θ is a compact ellipsoid in ℓ_2, that is,

$$\Theta = \{\theta : \sum a_i^2 \theta_i^2 \leq C_2, a_i > 0 \text{ and } a_i \to \infty\}.$$

Find the minimax linear risk $R_L(\Theta, \epsilon^2)$ under square loss.

[*Hint :* You need to use Kuhn-Tucker conditions (KKT conditions) to solve a constrained optimization problem. The final solutions is $\epsilon^2 \sum_i (1 - a_i/\mu)_+$ where μ is a number determined by some equation.]

3. Consider $x_1, \ldots, x_n$ a sample from the truncated normal distribution, with density

$$p(x \mid \theta) = \left(\frac{2}{\pi}\right)^{1/2} e^{-(x-\theta)^2/2} \mathbf{1}_{[\theta, +\infty]}(x).$$

Show that the best invariant estimator of θ under square loss is

$$\delta^*(x) = \bar{x} - \frac{\exp\{-n(x_{(1)} - \bar{x})^2/2\}}{\sqrt{2n\pi}\,\Phi(\sqrt{n}(x_{(1)} - \bar{x}))}.$$

4. Suppose $Y \sim f(y - \theta)$ (i.e. the distribution of Y is from a location family). A prior $\pi(\theta)$ is called *relative invariant* under location shift if

$$\pi(\theta + c) = \pi(\theta) w(c), \quad \forall \theta, c.$$

Show that the corresponding Bayes estimator $\delta^\pi(y)$ of θ (under square loss) is an invariant estimator.