Finding the SE of the Mean after Recentering the X values at c

Assume that we recenter our data by subtracting a constant, c, from all X_i's. Our new explanatory variable is now $Z_i = X_i - c$. (For the nematode example, $c = 5$.)

With the recentered data, what is the variance of Y given Z?

$$Var(\hat{\mu}\{Y|Z\}) = Var(\hat{\beta}_0 + \hat{\beta}_1 Z) \tag{1}$$

Recall $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{Z}$.

$$\begin{aligned} Var(\hat{\mu}\{Y|Z\}) &= Var(\overline{Y} - \hat{\beta}_1 \overline{Z} + \hat{\beta}_1 Z) && (2) \\ &= Var(\overline{Y} + \hat{\beta}_1 (Z - \overline{Z})) && (3) \\ &= Var(\overline{Y}) + (Z - \overline{Z})^2 Var(\hat{\beta}_1) + 2(Z - \overline{Z}) Cov(\overline{Y}, \hat{\beta}_1) && (4) \end{aligned}$$

Intuitively the average value, $\overline{Y}$, should not depend on the fitted slope, so $Cov(\overline{Y}, \hat{\beta}_1) = 0$.

$$\begin{aligned} Var(\hat{\mu}\{Y|Z\}) &= \frac{\sigma^2}{n} + (Z - \overline{Z})^2 \frac{\sigma^2}{(n-1)s_x^2} && (5) \\ &= \sigma^2 \left(\frac{1}{n} + \frac{(Z - \overline{Z})^2}{(n-1)s_x^2} \right) && (6) \end{aligned}$$

The square root of the above expression gives the standard deviation of the estimated mean of Y given Z. It is a function of n, s_x^2, σ^2 and $\overline{Z}$.

If we are using the centering trick, we are interested in the value of the mean response Y at $X = c$. Then $Z = 0$, and the formula for the SE for the mean Y at $Z = 0$ is identical to finding the SE of the intercept after recentering the data.

$$Var(\hat{\mu}\{Y|Z = 0\}) = \sigma^2 \left(\frac{1}{n} + \frac{\overline{Z}^2}{(n-1)s_x^2} \right) \tag{7}$$