Final Examination

STA 215: Statistical Inference

Saturday, 2001 May 5, 9:00am – 12:00 noon

This is an open-book examination, but you may not share materials. A normal distribution table, a PMF/PDF handout, and a blank worksheet are attached to the exam. If you don't understand something in one of the questions feel free to ask me, but please do not talk to each other. You may use a calculator but not a PDA or laptop computer.

You must **show** your **work** to get partial credit. Unsupported answers are not acceptable, even if they are correct. It is to your advantage to write your solutions as clearly as possible, since I cannot give credit for solutions I do not understand. Good luck.

Print Name:

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Problem 1: Let $X_j \sim \mathsf{Be}(\theta, 1)$ be independent beta-distributed random variables with density function

$$f(x \mid \theta) = \theta x^{\theta - 1}, \qquad 0 < x < 1,$$

for some $\theta \in \Theta = (0, \infty)$ and answer the following questions:

a. (5) Find the mean $\mu_X = \mathsf{E}[X \mid \theta]$ and variance $\sigma_X^2 = \mathsf{Var}[X \mid \theta]$:

 $\mu_X = _ \sigma_X^2 = _$

b. (5) Change variables to find the probability density function for $Y \equiv -\ln X$:

$$f_Y(y \mid \theta) = _$$

c. (5) Give the mean and variance for $Y \equiv -\ln X$:

 $\mu_Y = \underline{\qquad} \sigma_Y^2 = \underline{\qquad}$

d. (5) Find the Fisher Information I_X for a single observation of X: $I_X =$ _____ **Problem 2:** Again let $X_j \sim \mathsf{Be}(\theta, 1)$ be independent beta-distributed random variables with density function

$$f(x \mid \theta) = \theta x^{\theta - 1}, \qquad 0 < x < 1, \tag{1}$$

and let $\vec{x} = (x_1, ..., x_n) \in \mathcal{X} = (0, 1)^n$ be a random sample of size n:

a. (5) Find the likelihood function for θ upon observing $\vec{x} \in \mathcal{X}$.

 $L(\theta \mid \vec{x}) = _$

b. (15) If this is an Exponential Family, find the canonical parameter $\eta(\theta)$, the canonical sufficient statistic $T(\vec{x}) = T_n(\vec{x})$, and the normalizing function $B(\theta)$ in the Exponential Family representation

$$f(\vec{x} \mid \theta) = h_n(\vec{x})e^{\eta(\theta) \cdot T_n(\vec{x}) - n B(\theta)}$$

for a random sample of size n (you need not give $h_n(\vec{x}) = \prod h(x_j)$). If it is *not* an exponential family, explain why.

$$T_n(\vec{x}) = \underline{\qquad}$$
$$\eta(\theta) = \underline{\qquad}$$
$$B(\theta) = \underline{\qquad}$$

Problem 3: With the same Beta $\mathsf{Be}(\theta, 1)$ model as above, and with $\vec{x} = (x_1, ..., x_n) \in \mathcal{X} = (0, 1)^n$ a random sample of size n,

a. (10) Show that the Gamma $\theta \sim \mathsf{Ga}(\alpha, \beta)$ distribution with density

$$\pi(\theta) \equiv \beta^{\alpha} \, \theta^{\alpha-1} \, e^{-\beta \, \theta} / \Gamma(\alpha), \qquad \theta > 0$$

is conjugate for \vec{x} and find α^* , β^* (which may depend on α , β , n, and \vec{x}) such that $\pi(\theta \mid \vec{x}) \sim \mathsf{Ga}(\alpha^*, \beta^*)$:

$$\alpha^{\star} = \underline{\qquad} \qquad \beta^{\star} = \underline{\qquad}$$

b. (10) Find the Jeffreys prior density function $\pi_J(\theta)$ and give the Jeffreys posterior distribution (upon observing $\vec{x} \in \mathcal{X}$), either by giving its pdf or (better) by giving the distribution's name and parameters:

 $\pi_J(\theta) \propto _$ $\pi_J(\theta \mid \vec{x}) \sim _$

Problem 4: With the same Beta $Be(\theta, 1)$ model as above, and again with $\vec{x} = (x_1, ..., x_n) \in \mathcal{X} = (0, 1)^n$ a random sample of size n,

a. (5) Find the MLE $\hat{\theta}$:

 $\hat{\theta} =$ ______

b. (5) Find a Method of Moments (MOM) estimator $\tilde{\theta}$ (Hint: Solve 1(a) for θ as a function of μ_X , then substitute \bar{x} for μ_X):

 $\tilde{\theta} =$ ______

c. (5) Find the posterior mean $\bar{\theta}$ for a Bayesian analysis with Gamma prior distribution $\theta \sim \mathsf{Ga}(\alpha, \beta)$:

 $\bar{\theta} =$

d. (5) The Raô-Blackwell Theorem says that one of these three estimators could be improve to reduce its risk. Which one, and how does Raô-Blackwell suggest it may be improved? Be specific (use 2b!), but no calculations are needed. **Problem 5:** With the same Beta $Be(\theta, 1)$ model as above, and with n = 10 observations $\vec{x} = (x_1, ..., x_{10})$ satisfying

$$S_{1} \equiv \sum_{j=1}^{n} x_{j} = 6.389 | S_{2} \equiv \sum_{j=1}^{n} x_{j}^{2} = 5.088$$

$$S_{3} \equiv \sum_{j=1}^{n} \ln(x_{j}) = -6.384 | S_{4} \equiv \sum_{j=1}^{n} 1/x_{j} = 24.740$$

$$S_{5} \equiv \min_{1 \le j \le n} x_{j} = 0.159 | S_{6} \equiv \max_{1 \le j \le n} x_{j} = 0.973$$

a. (10) Find an expression for the Jeffreys posterior probability of the hypothesis H_0 : [$\theta \leq 1$]. Give the answer as your choice of either S-plus or Mathematica commands or as an explicit one-dimensional integral expression:

$$\pi_J(H_0 \mid \vec{x}) = _$$

b. (10) Perform a significance test of the hypothesis $H_0 : [\theta \leq 1]$ against the alternative $H_1 : [\theta > 1]$. Give the *p*-value as your choice of either S-plus or Mathematica commands or an explicit one-dimensional integral expression (Hint: what is the distribution of your sufficient statistic T_n from 2b?)

P = _____

Problem 6: Ron thinks that the recorded failure times t_j of felt dryboard marking pens have an Ex(1) distribution, with mean one hour, while Tom thinks they have a Ga(2,2) distribution, also with mean one hour. Consider the following statistics, each based on an independent sample $\vec{t} = \{t_j\}$ of size n:

$$\begin{array}{l} S_1 = \sum_{j=1}^n t_j \\ S_4 = \sum_{j=1}^n 1/t_j \end{array} \begin{vmatrix} S_2 = \sum_{j=1}^n t_j^2 \\ S_5 = \min_{1 \le j \le n} t_j \end{vmatrix} \begin{vmatrix} S_3 = \sum_{j=1}^n \ln(t_j) \\ S_6 = \max_{1 \le j \le n} t_j \end{vmatrix}$$

(20) How should Ron and Tom resolve their dispute? Suggest and defend a **statistic** based on one or more of the statistics $S_k = S_k(\vec{t}_n)$ above and a procedure for deciding whether $t_j \sim \mathsf{Ex}(1)$ or $t_j \sim \mathsf{Ga}(2,2)$. Describe briefly what computation would be required. Please present **both** a (10) Frequentist and a (10) Bayesian solution.

Problem X: Extra Credit:

a) (+5) (Problem 6, revisited). Use a normal approximation if necessary to complete the analysis (either one) you began in Problem 6; decide whether Ron or Tom is correct, and defend your choice, based on the following data for n = 100 observations:

$$S_{1} \equiv \sum_{j=1}^{n} t_{j} = 91.148 | S_{2} \equiv \sum_{j=1}^{n} t_{j}^{2} = 120.852 S_{3} \equiv \sum_{j=1}^{n} \ln(t_{j}) = -34.276 | S_{4} \equiv \sum_{j=1}^{n} 1/t_{j} = 194.509 S_{5} \equiv \min_{1 \le j \le n} t_{j} = 0.081 | S_{6} \equiv \max_{1 \le j \le n} t_{j} = 4.142$$

b) (+5) (Problem 6, revisited). Set $f_0(t) = e^{-t}$ and $f_1(t) = 4t e^{-2t}$ for t > 0, the density functions Ron and Tom believe govern $\{t_j\}$ in Problem 6. One can compute

$$K(f_0, f_1) \equiv \int_0^\infty \ln \frac{f_0(t)}{f_1(t)} f_0(t) dt = -2\ln 2 - \psi(1) + 1 \approx 0.190921$$

$$K(f_1, f_0) \equiv \int_0^\infty \ln \frac{f_1(t)}{f_0(t)} f_1(t) dt = \ln 2 + \psi(2) - 1 \approx 0.115932$$

Approximately how large a sample would be required to resolve Ron and Tom's debate (at least, to be 99% sure), and why?

c) (+0) What are Ron and Tom's *last* names?

Name: ____

Extra worksheet, if needed:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-t^{2}/2} dt$$

Table 5.1Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	x	.00	.01	$\frac{2}{.02}$.03	.04	.05	.06	.07	.08	.09
1 5398 5438 5517 5557 5596 5636 5675 5714 5753 2 5793 5832 5871 5910 5948 5987 6026 6064 6103 6111 3 6179 6217 6255 6293 6331 6368 6406 6443 6480 6517 4 6554 6591 6695 7019 7054 7088 7123 7157 7190 7224 6 7257 7291 7724 7737 7789 7744 7764 7764 7793 7764 7 7580 7110 7939 7967 7995 8023 8015 8078 8078 8078 830 8333 9 8153 8406 8438 8461 8485 8508 8531 8548 8507 8590 8611 8430 8807 8907 8912 8911 9141 9141 9161 9113 9117	_	.5000					.5199				
.8 .6179 .6217 .6255 .6293 .6331 .6368 .6406 .6443 .6480 .6517 .4 .6554 .6591 .6628 .6664 .6700 .6736 .6772 .6808 .6844 .6879 .5 .6915 .6950 .6985 .7019 .7054 .7088 .7123 .7157 .7190 .7224 .6 .7257 .7291 .7324 .7357 .7389 .7422 .7454 .7486 .7517 .7549 .7 .7580 .7611 .7642 .7673 .7704 .7734 .7744 .7744 .7823 .7852 .8 .7881 .7910 .7937 .8203 .8315 .8310 .8365 .8389 .0 .8413 .8461 .8485 .8508 .8511 .8554 .8577 .8591 .8512 .843 .869 .8888 .807 .8922 .8944 .8901 .9015 .9117 .914<											
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.4 .6554 .6591 .6628 .6664 .6700 .6736 .6772 .6808 .6844 .6879 .5 .6915 .6950 .6985 .7019 .7054 .7088 .7123 .7157 .7190 .7224 .6 .7257 .7291 .7324 .7357 .7389 .7422 .7454 .7486 .7517 .7549 .7 .7580 .7611 .7642 .7673 .7704 .7734 .7764 .7744 .7823 .7852 .8 .7881 .7910 .7939 .7967 .7995 .8023 .8051 .8078 .8106 .8133 .9 .8159 .8186 .8212 .8238 .8264 .8289 .8315 .8340 .8365 .8389 1.0 .8413 .8463 .8461 .8485 .8508 .8531 .8574 .8777 .8599 .8621 1.1 .8643 .8665 .8688 .8907 .9225 .9430 .9417 .9162 .9177 1.4 .9192 .9207 <	.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.6 .7257 .7291 .7324 .7357 .7389 .7422 .7454 .7486 .7517 .7549 .7 .7580 .7611 .7642 .7673 .7704 .7764 .7764 .7794 .7823 .7852 .8 .7881 .7910 .7939 .7967 .7995 .8023 .8051 .8078 .8106 .8133 .9 .8159 .8186 .8212 .8238 .8264 .8289 .8315 .8340 .8365 .8389 1.0 .8413 .8438 .8461 .8485 .8508 .8511 .8554 .8577 .8599 .8621 1.1 .8643 .8665 .8686 .8708 .8729 .8770 .8700 .810 .8810 1.2 .849 .8669 .8888 .8907 .9215 .9131 .9147 .9162 .9177 1.4 .9192 .9207 .9222 .9236 .9255 .9515 .9525 .95	.4	.6554	.6591	.6628		.6700	.6736	.6772	.6808	.6844	.6879
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1.5.9332.9345.9357.9370.9382.9394.9406.9418.9429.94411.6.9452.9463.9474.9484.9495.9505.9515.9525.9535.95451.7.9554.9564.9573.9582.9591.9599.9608.9616.9625.96331.8.9641.9649.9656.9664.9671.9678.9686.9693.9699.97061.9.9713.9719.9726.9732.9738.9744.9750.9756.9761.97672.0.9772.9778.9783.9788.9793.9798.9803.9808.9812.98172.1.9821.9826.9830.9834.9838.9842.9846.9850.9854.98572.2.9861.9864.9868.9871.9875.9878.9881.9844.9877.99162.3.9893.9896.9898.9901.9904.9906.9909.9911.9913.99162.4.9918.9920.9925.9927.9929.931.9322.9934.99362.5.9938.9940.9917.9913.9916.9962.9963.99642.4.9918.9920.9927.9929.9931.9922.9933.99642.5.9938.9946.9945.9960.9961.9962.9963.99642.6.9953.9955	1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.6 $.9452$ $.9463$ $.9474$ $.9484$ $.9495$ $.9505$ $.9515$ $.9525$ $.9535$ $.9545$ 1.7 $.9554$ $.9564$ $.9573$ $.9582$ $.9591$ $.9599$ $.9608$ $.9616$ $.9625$ $.9633$ 1.8 $.9641$ $.9649$ $.9656$ $.9664$ $.9671$ $.9678$ $.9686$ $.9693$ $.9699$ $.9706$ 1.9 $.9713$ $.9719$ $.9726$ $.9732$ $.9738$ $.9744$ $.9750$ $.9756$ $.9761$ $.9767$ 2.0 $.9772$ $.9778$ $.9783$ $.9788$ $.9793$ $.9798$ $.9803$ $.9808$ $.9812$ $.9817$ 2.1 $.9821$ $.9826$ $.9830$ $.9834$ $.9838$ $.9842$ $.9846$ $.9850$ $.9854$ $.9857$ 2.2 $.9861$ $.9864$ $.9868$ $.9871$ $.9875$ $.9878$ $.9881$ $.9884$ $.9887$ $.9890$ 2.3 $.9893$ $.9896$ $.9898$ $.9901$ $.9906$ $.9909$ $.9911$ $.9913$ $.9916$ 2.4 $.9918$ $.9920$ $.9922$ $.9925$ $.9927$ $.9929$ $.9331$ $.9932$ $.9934$ $.9936$ 2.4 $.9918$ $.9920$ $.9925$ $.9927$ $.9929$ $.9931$ $.9932$ $.9934$ $.9936$ 2.4 $.9918$ $.9940$ $.9941$ $.9943$ $.9946$ $.9948$ $.9949$ $.9951$ $.9952$ 2.6 $.9955$ $.9956$ $.9977$ $.$	1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.7 $.9554$ $.9564$ $.9573$ $.9582$ $.9591$ $.9599$ $.9608$ $.9616$ $.9625$ $.9633$ 1.8 $.9641$ $.9649$ $.9656$ $.9664$ $.9671$ $.9678$ $.9686$ $.9693$ $.9699$ $.9706$ 1.9 $.9713$ $.9719$ $.9726$ $.9732$ $.9738$ $.9744$ $.9750$ $.9756$ $.9761$ $.9767$ 2.0 $.9772$ $.9778$ $.9783$ $.9788$ $.9793$ $.9798$ $.9803$ $.9808$ $.9812$ $.9817$ 2.1 $.9821$ $.9826$ $.9830$ $.9834$ $.9838$ $.9842$ $.9846$ $.9850$ $.9854$ $.9857$ 2.2 $.9861$ $.9864$ $.9868$ $.9871$ $.9875$ $.9878$ $.9881$ $.9884$ $.9877$ $.9890$ 2.3 $.9893$ $.9896$ $.9898$ $.9001$ $.9044$ $.9906$ $.9909$ $.9911$ $.9913$ $.9916$ 2.4 $.9918$ $.9920$ $.9922$ $.9927$ $.9929$ $.9931$ $.9932$ $.9934$ $.9936$ 2.5 $.9938$ $.9940$ $.9941$ $.9943$ $.9946$ $.9948$ $.9949$ $.9951$ $.9952$ 2.6 $.9953$ $.9955$ $.9956$ $.9957$ $.9957$ $.9970$ $.9971$ $.9972$ $.9973$ $.9974$ 2.7 $.9965$ $.9966$ $.9967$ $.9977$ $.9977$ $.9978$ $.9979$ $.9980$ $.9981$ 2.7 $.9965$ $.9966$ $.9977$ $.$	1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.8 .9641 .9649 .9656 .9664 .9671 .9678 .9686 .9693 .9699 .9706 1.9 .9713 .9719 .9726 .9732 .9738 .9744 .9750 .9756 .9761 .9767 2.0 .9772 .9778 .9783 .9788 .9793 .9798 .9803 .9808 .9812 .9817 2.1 .9821 .9826 .9830 .9834 .9838 .9842 .9846 .9850 .9854 .9857 2.2 .9861 .9864 .9868 .9871 .9875 .9878 .9881 .9844 .9887 .9890 2.3 .9893 .9896 .9898 .9001 .9044 .9006 .9099 .9911 .9913 .9913 2.4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .9936 2.4 .9918 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953	1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	
1.9.9713.9719.9726.9732.9738.9744.9750.9756.9761.97672.0.9772.9778.9783.9788.9793.9798.9803.9808.9812.98172.1.9821.9826.9830.9834.9838.9842.9846.9850.9854.98572.2.9861.9864.9868.9871.9875.9878.9881.9884.9887.98902.3.9893.9896.9898.901.9904.9906.9909.9911.9913.99162.4.9918.9920.9922.9925.9927.9929.9931.9932.9934.99362.5.9938.9940.9941.9943.9945.9946.9948.9949.9951.99522.6.9953.9955.9956.9957.9959.9960.9961.9962.9963.99642.7.9965.9966.9967.9968.9970.9971.9972.9973.99742.8.9974.9975.9976.9977.9978.9979.9980.99812.9.9981.9982.9983.9984.9984.9985.9986.99863.0.9987.9987.9987.9988.9989.9989.9990.99903.1.9990.9991.9991.9992.9992.9992.9993.99933.2.9933.9994.9994.9994	1.7	.9554	.9564	.9573	.9582		.9599	.9608	.9616	.9625	.9633
2.0 .9772 .9778 .9783 .9788 .9793 .9798 .9803 .9808 .9812 .9817 2.1 .9821 .9826 .9830 .9834 .9838 .9842 .9846 .9850 .9854 .9857 2.2 .9861 .9864 .9868 .9871 .9875 .9878 .9881 .9884 .9887 .9890 2.3 .9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 .9916 2.4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .9936 2.5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 .9962 .9963 .9964 2.7 .9965 .9966 .9967 .9968 .9970 .9971 .9972 .9973 .9974	1.8		.9649				.9678		.9693		
2.1 .9821 .9826 .9830 .9834 .9838 .9842 .9846 .9850 .9854 .9857 2.2 .9861 .9864 .9868 .9871 .9875 .9878 .9881 .9884 .9887 .9890 2.3 .9893 .9896 .9898 .901 .904 .906 .9099 .911 .913 .916 2.4 .9918 .9920 .9922 .9925 .9927 .9929 .931 .932 .934 .9936 2.5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 2.7 .9965 .9966 .9957 .9959 .9960 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9976 .9977 .9978 .9979 .9980 .9980 .9981 2.9 .9981 .9982 .9983		.9713	.9719	.9726		.9738	.9744	.9750	.9756		.9767
2.2 .9861 .9864 .9868 .9871 .9875 .9878 .9881 .9844 .9877 .9890 2.3 .9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 .9916 2.4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .9936 2.5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 2.7 .9965 .9966 .9967 .9959 .9960 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9976 .9977 .9978 .9979 .9979 .9980 .9981 2.9 .9981 .9982 .9983 .9984 .9985 .9986 .9986 .9986 3.0 .9987 .9987 .9987 .9988 .9989		.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.3 .9893 .9896 .9898 .9901 .9904 .9906 .9909 .9911 .9913 .9916 2.4 .9918 .9920 .9922 .9925 .9927 .9929 .9931 .9932 .9934 .9936 2.5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 2.7 .9965 .9966 .9967 .9968 .9970 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9976 .9977 .9978 .9979 .9980 .9981 2.9 .9981 .9982 .9983 .9984 .9985 .9986 .9986 .9986 2.9 .9987 .9987 .9987 .9988 .9984 .9985 .9986 .9990 .9990 3.0 .9987 .9987 .9987 .9981 .9992 .9992 .9992			.9826	.9830		.9838			.9850		.9857
2.4 .9918 .9920 .9922 .9927 .9929 .9931 .9932 .9934 .9936 2.5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 2.7 .9965 .9966 .9967 .9968 .9970 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9976 .9977 .9978 .9979 .9979 .9980 .9981 2.9 .9981 .9982 .9983 .9984 .9985 .9985 .9986 .9986 3.0 .9987 .9987 .9988 .9988 .9989 .9989 .9990 .9990 3.1 .9990 .9991 .9991 .9992 .9992 .9992 .9993 .9993 .9993 3.2 .9993 .9994 .9994 .9994 .9994 .9996 .9995 .9995 .9995			.9864	.9868		.9875	.9878	.9881	.9884		.9890
2.5 .9938 .9940 .9941 .9943 .9945 .9946 .9948 .9949 .9951 .9952 2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 2.7 .9965 .9966 .9967 .9969 .9970 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9976 .9977 .9978 .9979 .9979 .9980 .9981 2.9 .9981 .9982 .9983 .9984 .9985 .9985 .9986 .9986 3.0 .9987 .9987 .9988 .9988 .9989 .9989 .9990 .9990 3.1 .9990 .9991 .9991 .9992 .9992 .9992 .9993 .9993 3.2 .9993 .9994 .9994 .9994 .9994 .9996 .9995 .9995 .9995 3.3 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996	2.3	.9893	.9896	.9898		.9904	.9906	.9909	.9911	.9913	.9916
2.6 .9953 .9955 .9956 .9957 .9959 .9960 .9961 .9962 .9963 .9964 2.7 .9965 .9966 .9967 .9968 .9969 .9970 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9976 .9977 .9978 .9979 .9979 .9980 .9981 2.9 .9981 .9982 .9983 .9984 .9985 .9985 .9986 .9986 3.0 .9987 .9987 .9987 .9988 .9988 .9989 .9989 .9990 .9990 .9990 .9991 .9991 .9992 .9992 .9993 .9993 .9993 .9993 .9993 .9994 .9994 .9994 .9994 .9994 .9995 .9995 .9995 .9995 .9995 .9996			.9920	.9922		.9927	.9929	.9931	.9932	.9934	.9936
2.7 .9965 .9966 .9967 .9968 .9969 .9970 .9971 .9972 .9973 .9974 2.8 .9974 .9975 .9976 .9977 .9978 .9979 .9979 .9980 .9981 2.9 .9981 .9982 .9982 .9983 .9984 .9985 .9985 .9986 .9986 3.0 .9987 .9987 .9987 .9988 .9988 .9989 .9989 .9990 .9990 .9990 3.1 .9990 .9991 .9991 .9991 .9994 .9992 .9992 .9992 .9993 .9993 .9995 3.2 .9993 .9993 .9994 .9994 .9994 .9994 .9994 .9995 .9995 .9995 3.3 .9995 .9995 .9995 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9996 .9997											
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	3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998