

STAT215: Homework 2

Due: Wednesday, Feb 15

1. (10 pt) Let X_1, X_2, \dots, X_n be a sample from a population with double exponential pdf:

$$f(x|\theta) = \frac{1}{2}e^{-|x-\theta|}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty.$$

Find the MLE of θ . (Hint: consider order statistics.)

2. (15 pt) Let X_1, X_2, \dots, X_n be iid with pdf:

$$f(x|\theta) = \theta x^{\theta-1}, \quad 0 \leq x \leq 1, \quad 0 < \theta < \infty.$$

Find the MLE of θ , and show that its variance $\rightarrow 0$ as $n \rightarrow \infty$.

3. (15 pt) Suppose that the random variables Y_1, \dots, Y_n satisfy

$$Y_i = \beta x_i + \epsilon_i, \quad i = 1, \dots, n$$

where x_1, \dots, x_n are fixed constants, and $\epsilon_1, \dots, \epsilon_n$ are iid $N(0, \sigma^2)$, σ^2 unknown.

- (a) Find a two-dimensional sufficient statistic for (β, σ^2) .
 - (b) Find the MLE of β , and show that it is an unbiased estimator of β .
 - (c) Find the distribution of the MLE of β .
4. (10 pt each) Bickel & Doksum problems from pages 142-152: 2.2.5, 2.2.10
5. (10 pt each) Bickel & Doksum problems from pages 152-154: 2.3.7
6. (10 pt each) Bickel & Doksum problems from pages 197-199: 3.2.2, 3.2.4, 3.2.8
7. (10 pt) Bonus problem:
Bickel & Doksum problems: 2.2.21