STAT215: Homework 4

Due: Wednesday, April 5

1. (10 pt) Find the LRT (Likelihood Ratio Test) of
$$H_0 : \theta \leq 0 \quad \text{versus} \quad H_1 : \theta > 0$$
based on a sample $\{X_1, \cdots, X_n\}$ of size $n \in \mathbb{N}$ from a population with probability density function
$$f(x|\theta, \lambda) = \lambda e^{-\lambda(x-\theta)} I_{[\theta,\infty)}(x), \quad \theta \in \mathbb{R}, \quad \lambda \in \mathbb{R}^+$$
where both θ and λ are unknown.

2. (10 pt) A family of probability density functions $f(x \mid \theta)$ on $\mathbb{R}$, indexed by $\theta \in \Theta \subset \mathbb{R}$, is said to have a *monotone likelihood ratio* (MLR) if, for each $\theta_0 \neq \theta_1$, the ratio $f(x \mid \theta_1)/f(x \mid \theta_0)$ is monotonic in x. Consider the Cauchy scale family with pdfs
$$f(x|\theta) = \frac{\theta/\pi}{\theta^2 + x^2}, \quad -\infty < x < \infty, \quad \theta > 0$$
   (a) Show that this family does not have an MLR.
   (b) If X is one observation from $f(x|\theta)$, show that $|X|$ is sufficient for θ and that the distribution of $|X|$ does have an MLR.

3. (20 pt) A family of probability distributions on $\mathbb{R}$, indexed by $\theta \in \Theta \subset \mathbb{R}$, is called *stochastically increasing* if their CDFs $\{F(x \mid \theta),\ \theta \in \Theta\}$ satisfy
$$\theta_1 < \theta_2 \Longrightarrow F(x \mid \theta_1) \geq F(x \mid \theta_2) \quad \forall x \in \mathbb{R}.$$
(Intuitively this says that larger values of the parameter θ are associated with larger values of the random variable X).
   (a) Show that if a family of pdfs $\{f(x|\theta) : \theta \in \Theta\}$ has an MLR, then the corresponding family of CDFs is stochastically increasing in θ.
   (b) Show that the converse of part (a) is false.

4. (10 pt) Bickel & Doksum, page 269: 4.1.1
5. (10 pt) Bickel & Doksum, page 270: 4.1.3
6. (10 pt) Bickel & Doksum, page 272: 4.2.3
7. (10 pt) Bickel & Doksum, page 276: 4.3.9
8. (10 pt) Bickel & Doksum, page 290: 4.9.1
9. (10 pt) Bickel & Doksum, page 291: 4.9.2