Final Examination

Mth 136 = Sta 114

Monday, 2009 April 27, 2:00 – 5:00pm

- This is a **closed book** exam— please put your books on the floor.
- You may use a calculator and **two pages** of your own notes.
  Do not share calculators or notes.
- Please ask me questions if a problem needs clarification.
- **Show** your **work**. Boxing answers helps me find them.
- Numerical answers: **four significant digits** or fractions in lowest terms. **Simplify** expressions for full credit.
- Distribution and pdf/pmf tables are attached.

Cheating on exams is a breach of trust with classmates and faculty that will not be tolerated. After completing the exam please acknowledge that you have adhered to the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

Signature: ______________________

Print Name: ______________________

1.	/20	5.	/20
2.	/20	6.	/20
3.	/20	7.	/20
4.	/20	8.	/20
Total:			/160

Problem 1: The number of successes in n independent trials, all with the same probability of success p, has the binomial distribution with

$$\mathsf{P}[X = x \mid p] = \binom{n}{x} p^x (1-p)^{n-x}, \quad 0 \le x \le n.$$

Consider a number of (perhaps medical) experiments, each consisting of $n = 10$ independent trials.

a) (5) Find the Maximum Likelihood Estimate (MLE) $\hat{p}(\mathbf{x})$ for p, if we observe the data set $\mathbf{x} = \{7, 5, 9, 6, 3\}$. Show your work— *derive* the result, don't just write down the answer.

$\hat{p} =$ ____________________

b) (5) Find a sufficient statistic $T(\mathbf{x})$ for m observations $x_1, ..., x_m$ from this distribution, and explain why T is sufficient.

$T =$ ____________________

Problem 1 (cont):

c) (5) For the data set $\mathbf{x}$ given above, an exact 90% Confidence Interval for p turns out to be: [0.47388, 0.71687] (you don't have to verify this). Exactly **what event** is it that has probability 0.90?

d) (5) Camden prefers to think about the odds $\theta = \frac{p}{1-p}$ and writes the binomial pmf in the form

$$\mathsf{P}[X = x \mid \theta] = \binom{n}{x}(1+\theta)^{-n}\theta^x, \quad 0 \le x \le n.$$

What will her MLE be, with the same data $\mathbf{x}$ as before? Why?
Hint: No new maximizing is needed...

$\hat{\theta} =$ ____________________

Problem 2: For 2pt each, circle "T" for True or "F" for False, or write short answers in the boxes. Questions marked "$\star$" concern a random sample $\mathbf{x} = \{X_i\}_{i \le N}$ from the $\mathsf{No}(\mu, \sigma^2)$ distribution. No explanations are required.

a) A P-value is the probability that H_0 is true. T F

b) What prior distribution is conjugate for Poisson data? [ ]

c) $\star$ If $\sigma^2 = 1$ and $N = 4$ and if μ has prior $\xi(\mu) \sim \mathsf{No}(0, 1)$ then what is the posterior distribution $\xi(\mu \mid \mathbf{x})$ for μ? [ ]

d) If a P-value satisfies $P > 0.99$ then Reject H_0 at $\alpha = 0.02$ T F

e) $\star$ Use a t distribution to estimate μ if σ^2 is unknown. T F

f) $\star$ Use a t distribution to estimate μ if $\sigma^2 = 1$ but N is small. T F

g) If $\{Y_i\} \sim \mathsf{Po}(\lambda)$ then $1/\bar{Y}$ is sufficient for λ. T F

h) The *power* of a test is the probability H_0 is false. T F

i) $\star$ The Generalized Likelihood Ratio Test (geneneralized LRT) of the two-sided hypothesis $H_0: \ \mu{=}0$ vs. $H_1: \ \mu{\neq}0$ is UMP. T F

j) $\star$ What is the distribution of $\sum_{i \le N} \frac{(X_i - \mu)^2}{\sigma^2}$ if $N = 10$? [ ]

Problem 3: The number of green markers used by a certain Statistics professor in a semester has a Poisson probability distribution with mean parameter λ,

$$\mathsf{P}[X = k \mid \lambda] = \frac{\lambda^k}{k!} e^{-\lambda}, \qquad k = 0, 1, ...$$

Corey and Chris, two students in his class, disagree about λ; Corey thinks it's $\lambda_0 = 4$, while Chris thinks $\lambda_1 = 5$. Unfortunately they have only one data point, from the class they shared, in which $x = 7$ green markers were used.

a) (5) If they choose to follow a Bayesian approach with equal prior probabilities $\xi_0 = \xi_1 = 1/2$ for their two favorite values of λ, what is the posterior probability that Corey is right? Give exact (simplified) answer, or four significant digits.a

$\xi[\lambda = \lambda_0 \mid x = 7] =$ ____________________

b) (5) If instead they choose to use sampling-based methods, give a sum or integral expression for calculating the P-value for the null hypothesis $H_0 : \lambda = 4$ against the alternative $H_1 : \lambda = 5$ (no need to find the exact numerical value, which happens to be $P = 0.1106740$).

Problem 3 (cont):

The Statistics professor still uses a Poisson-distributed number of green markers, with p.m.f.

$$\mathsf{P}[X = k \mid \lambda] = \frac{\lambda^k}{k!} e^{-\lambda}, \qquad k = 0, 1, \ldots$$

c) (5) If Corey and Chris agree that any value of $\lambda > 0$ is possible, and try to estimate λ using a Gamma $\mathsf{Ga}(9, 2)$ prior distribution with mean $9/2 = 4.5$ (half-way between their guesses), find the posterior distribution for λ with the single observation $x = 7$. (Hint: it will be one of the distributions in the attached table) Give either its density function *or* its name and parameter value(s). Also give its mean (simplified exact or 4 sig fig) as their posterior estimate of λ. Show your work. No integration is necessary.

$\xi(\lambda \mid x = 7) =$ ______________ $\mathsf{E}[\lambda \mid x = 7] =$ ______________

d) (5) Give a sum or integral expression for calculating the probability that Corey is closer to the right answer (no need to give the exact numerical answer of 0.2822072). Be explicit and simplify.
$\xi(\lambda \leq 4.5 \mid x = 7) =$

Problem 4: Each day the Acme Widget Company makes a batch of widgets (many thousands), which they sell by the box (a few hundred widgets per box). Widget weights $\{W_i\}$ (in grams) are well-known to have a normal probability distribution $W_i \sim \mathsf{No}(\mu, \sigma^2)$ with $\sigma^2 = 25\,\mathrm{g}^2$ (*you* knew that, didn't you?), but each day's batch has its own mean weight μ.

Chuck Norris is weighing widgets to prepare for his upcoming film *Widget Deathwatch*. From his vast past experience Chuck knows that batch means μ for widget weights seem to have a normal distribution (or prior, in this context) with mean $75\,\mathrm{g}$ and variance $100\,\mathrm{g}^2$.

Today FedEx just brought Chuck a new box of widgets from today's (huge and otherwise unobserved) batch. What a great day. Let μ be the mean widget weight (in grams) for today's batch.

a) (5) Before opening the box of widgets, what's the probability that μ exceeds $100\,\mathrm{g}$?

$\xi(\mu > 100) =$ ______________

b) (15) Eager Chuck breaks open the box (karate chop) and puts 100 widgets on the scale; their total weight is $10.5\,\mathrm{kg}$. Now what is the probability that μ exceeds $100\,\mathrm{g}$? Show your work.

$\xi\big(\mu > 100 \mid \sum_{i=1}^{100} W_i = 10500\big) =$ ______________

Problem 5: The distances $\{X_i\}_{i\le N}$ (in furlongs[1]) of Barry Bonds' booming baseball blows[2] in batting practice are modeled as independent draws from the $\mathsf{Ex}(\lambda)$ distribution, with density function

$$f_X(x \mid \lambda) = \lambda e^{-\lambda x}\mathbf{1}_{\{\lambda>0\}}.$$

We'd like to learn about λ— unfortunately, we're outside the stadium and don't get to observe any of the X_i's directly. We only get to see how many of the hits go over the fence, exactly 1.0 furlong from home plate— *i.e.*, we only get to observe the indicator random variables

$$Y_i = \mathbf{1}_{\{X_i>1\}} = \begin{cases} 1 & X_i > 1 \\ 0 & X_i \le 1. \end{cases}$$

a) (5) What is the probability distribution for Y_1, as a function of λ? Give name and parameter(s), or pmf/pdf. Simplify!

   $\mathsf{P}[Y_1 = y \mid \lambda] = \Bigg\{$

b) (5) Give the MLE for λ upon observing $\mathbf{y} = \{Y_i\}_{i\le N}$ with $N = 100$ observations, which included 25 ones and 75 zeros:

   $\hat\lambda(\mathbf{y}) =$ ____________________

[1] A "furlong" is one eighth of a mile, 660 feet, or 201.168 m. Pretty far to hit a baseball.
[2] *hits*, really, but I was swept up in the alliteration

Problem 5 (cont):

Recall $Y_i = \mathbf{1}_{\{X_i>1\}}$ with $\{X_i\}_{i\le N} \overset{\text{iid}}{\sim} \mathsf{Ex}(\lambda)$.

c) (2) In what units is λ measured?

d) (8) Use a normal approximation to find the approximate P-value for a test of the hypotheses

$$H_0: \ \lambda = 2 \qquad vs. \qquad H_1: \ \lambda = 1$$

for these data. [First you have to decide *what outcomes* are more extreme, and *what variable* has approximately a normal distribution.]

$P =$ ____________________

Problem 6: If $\{X_i\}$, $1 \le i \le 10$ are sunburn skin temperature measurements for the left arms of 10 volunteer subjects, all prepared with a newly-formulated SPF-99 sunburn protection cream intended to reduce skin temperatures, and if $\{Y_i\}$, $1 \le i \le 10$ are sunburn temperature measurements for the same subjects' right arms, all treated with Crisco vegetable shortening (so, for example, X_7 and Y_7 are the two measurements from subject #7, named Septimus), and if we agree to the model that all measurements follow normal distributions with the same (unknown) variance but possibly different means μ_x and μ_y, then:

a) (5) To test to see if the SPF-99 cream is effective or not, what would be the null hypothesis, and what would be the alternative?[3]

H_0 : H_1 :

b) (5) Which test procedure would be best, and why?

○ Paired t ○ Unpaired t ○ Normal (z) ○ χ^2

[3] Remember— hypotheses are statements about *probability distributions* (or their parameters), not about "effectiveness" and not about data.

Problem 6 (cont):

Here are some statistics from the sunburn experiment:

$$\begin{aligned} \sum X_i &= 100 & \sum (X_i - \bar{X})^2 &= 428 \\ \sum Y_i &= 120 & \sum (Y_i - \bar{Y})^2 &= 220 \\ \sum [X_i - Y_i] &= -20 & \sum \left([X_i - Y_i] - \overline{[X - Y]}\right)^2 &= 72 \end{aligned}$$

c) (10) Test the hypothesis you chose in a), and evaluate the test statistic for the test procedure you chose in b), using these data. Should you accept or reject H_0 (choose and specify any necessary criteria)? Why?

Problem 7: Gregor Mendel is up to his tricks again, observing the shape and color of peas that resulted from certain crossbreedings. The reported results of one such experiment were:

round yellow	315	wrinkled yellow	101
round green	108	wrinkled green	32

(10) This time he wants to test the hypothesis that color (yellow *vs.* green) and smoothness (round *vs.* wrinkled) are independent traits (not necessarily with relative rates 9:3:3:1— it could be 15:5:3:1, for example). Find the test statistic, degrees of freedom, and P-value. Briefly explain how can you use one of the table*s* attached to this exam to find the P-value to at least two significant digits, even though the χ^2 table doesn't go down that low.

$Q =$ ______________ d.f. = ______________ $P =$ ______________

(10) A casino tested a new supplier's dice, counting the number of times each face appeared as a mechanical dice-roller repeatedly tossed them. The results from 1200 rolls of one die were:

1	2	3	4	5	6
220	185	190	175	200	230

Would you accept or reject the hypothesis that the die is fair, at level $\alpha = 0.05$? At level $\alpha = 0.01$? Show your work.

Problem 8: A series of Statistics students answer (independently, of course) the question "Was your statistics final too long?" We record the number X of consecutive "No, it was fine" answers before the first "Well, I thought it was a little too long." Of course X has the Geometric Distribution, with probability mass function

$$f(x \mid p) = \mathsf{P}[X = x \mid p] = p\, q^x, \qquad x = 0, 1, 2, ...$$

We wish to test the hypothesis $H_0:\ p = 1/2$ about the probability p of a "Too long" answer. If H_0 were true, then "Yes" and "No" answers would be like Heads and Tails of a fair coin. Give all answers **exactly** and in as **simple** a form as you can. For parts a) and b) we observe $X = 3$:

a) (5) Find the P-value for $H_0 : p = 1/2$ vs. $H_1 : p < 1/2$, if $X = 3$ [First decide what outcomes are more extreme for this H_1]:

$P =$ ______________

b) (5) Now for $H_0 : p = 1/2$ vs. the other one-sided alternative $H_1 : p > 1/2$, if $X = 3$:

$P =$ ______________

Problem 8 (cont):

Now let's be more optimistic and suppose we observe $X = 10$:

c) (2) For $H_0 : p = 1/2$ vs. $H_1 : p < 1/2$, if we observe $X = 10$:

$P =$ ______________

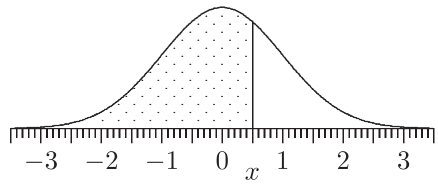
d) (8) With a uniform prior distribution with density $\xi(p) = \mathbf{1}_{\{0<p<1\}}$, what is the posterior expectation of p if we observe $X = 10$?

$\mathsf{E}_\xi(p \mid x = 10) =$ ______________

Done! Have a great summer.

Extra worksheet, if needed:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}\, dz:$$



Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

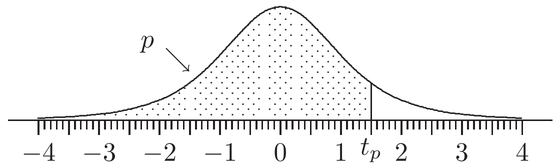
x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$

$\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$

Critical Values for Student's t

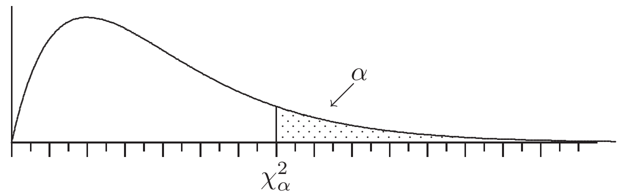
$$p = \int_{-\infty}^{t_p} c \, \frac{dt}{(1+t^2/\nu)^{(\nu+1)/2}}$$



ν	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$	$t_{.9999}$
1	0.325	0.727	1.376	1.9626	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.3862	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.2498	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.1896	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.1558	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.1342	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.1192	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.1081	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.0997	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.0931	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.0877	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.0832	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.0795	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.0763	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.0735	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.0711	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.0690	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.0672	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.0655	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.0640	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.0627	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.0614	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.0603	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.0593	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.0584	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.0575	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.0567	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.0560	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.0553	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.0547	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.0500	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.0455	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.0409	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.0364	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

Critical Values for χ^2

$$\alpha = \int_{\chi^2_\alpha}^{\infty} c\, x^{\nu/2-1} e^{-x/2}\, dx$$



ν	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	$\chi^2_{.001}$	$\chi^2_{.0005}$	$\chi^2_{.0001}$
1	0.4549	1.3233	2.7055	3.8415	5.0239	6.6349	7.87940	10.8276	12.1157	15.1367
2	1.3863	2.7726	4.6052	5.9915	7.3778	9.2103	10.5966	13.8155	15.2018	18.4207
3	2.3660	4.1083	6.2514	7.8147	9.3484	11.3449	12.8382	16.2662	17.7300	21.1075
4	3.3567	5.3853	7.7794	9.4877	11.1433	13.2767	14.8603	18.4668	19.9974	23.5127
5	4.3515	6.6257	9.2364	11.0705	12.8325	15.0863	16.7496	20.5150	22.1053	25.7448
6	5.3481	7.8408	10.6446	12.5916	14.4494	16.8119	18.5476	22.4577	24.1028	27.8563
7	6.3458	9.0371	12.0170	14.0671	16.0128	18.4753	20.2777	24.3219	26.0178	29.8775
8	7.3441	10.219	13.3616	15.5073	17.5345	20.0902	21.9550	26.1245	27.8680	31.8276
9	8.3428	11.389	14.6837	16.9190	19.0228	21.6660	23.5894	27.8772	29.6658	33.7199
10	9.3418	12.549	15.9872	18.3070	20.4831	23.2092	25.1882	29.5883	31.4198	35.5640
11	10.341	13.701	17.2750	19.6751	21.9200	24.7249	26.7568	31.2641	33.1366	37.3670
12	11.340	14.845	18.5493	21.0260	23.3366	26.2169	28.2995	32.9095	34.8213	39.1344
13	12.340	15.984	19.8119	22.3620	24.7356	27.6882	29.8195	34.5282	36.4778	40.8707
14	13.339	17.117	21.0641	23.6848	26.1189	29.1412	31.3193	36.1233	38.1094	42.5793
15	14.339	18.245	22.3071	24.9958	27.4884	30.5779	32.8013	37.6973	39.7188	44.2632
16	15.338	19.369	23.5418	26.2962	28.8453	31.9999	34.2672	39.2524	41.3081	45.9249
17	16.338	20.489	24.7690	27.5871	30.1910	33.4087	35.7185	40.7902	42.8792	47.5664
18	17.338	21.605	25.9894	28.8693	31.5264	34.8053	37.1565	42.3124	44.4338	49.1894
19	18.338	22.718	27.2036	30.1435	32.8523	36.1909	38.5823	43.8202	45.9731	50.7955
20	19.337	23.828	28.4120	31.4104	34.1696	37.5662	39.9968	45.3147	47.4985	52.3860
21	20.337	24.935	29.6151	32.6706	35.4789	38.9322	41.4011	46.7970	49.0108	53.9620
22	21.337	26.039	30.8133	33.9244	36.7807	40.2894	42.7957	48.2679	50.5111	55.5246
23	22.337	27.141	32.0069	35.1725	38.0756	41.6384	44.1813	49.7282	52.0002	57.0746
24	23.337	28.241	33.1962	36.4150	39.3641	42.9798	45.5585	51.1786	53.4788	58.6130
25	24.337	29.339	34.3816	37.6525	40.6465	44.3141	46.9279	52.6197	54.9475	60.1403
26	25.336	30.435	35.5632	38.8851	41.9232	45.6417	48.2899	54.0520	56.4069	61.6573
27	26.336	31.528	36.7412	40.1133	43.1945	46.9629	49.6449	55.4760	57.8576	63.1645
28	27.336	32.620	37.9159	41.3371	44.4608	48.2782	50.9934	56.8923	59.3000	64.6624
29	28.336	33.711	39.0875	42.5570	45.7223	49.5879	52.3356	58.3012	60.7346	66.1517
30	29.336	34.800	40.2560	43.7730	46.9792	50.8922	53.6720	59.7031	62.1619	67.6326
40	39.336	45.616	51.8051	55.7585	59.3417	63.6907	66.7660	73.4020	76.0946	82.0623
50	49.335	56.334	63.1671	67.5048	71.4202	76.1539	79.4900	86.6608	89.5605	95.9687
60	59.335	66.981	74.3970	79.0819	83.2977	88.3794	91.9517	99.6072	102.695	109.503
70	69.335	77.577	85.5270	90.5312	95.0232	100.425	104.215	112.317	115.578	122.755
80	79.334	88.130	96.5782	101.879	106.629	112.329	116.321	124.839	128.261	135.782
90	89.334	98.650	107.565	113.145	118.136	124.116	128.299	137.208	140.782	148.627
100	99.334	109.14	118.498	124.342	129.561	135.807	140.169	149.449	153.167	161.319

Name	Notation	pdf/pmf	Range	Mean μ	Variance σ^2	
Beta	$\mathsf{Be}(\alpha,\beta)$	$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$x\in(0,1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Binomial	$\mathsf{Bi}(n,p)$	$f(x)=\binom{n}{x}p^x q^{(n-x)}$	$x\in 0,\cdots,n$	$n\,p$	$n\,p\,q$	$(q=1-p)$
Exponential	$\mathsf{Ex}(\lambda)$	$f(x)=\lambda\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$	
Gamma	$\mathsf{Ga}(\alpha,\lambda)$	$f(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	α/λ	α/λ^2	
Geometric	$\mathsf{Ge}(p)$	$f(x)=p\,q^x$	$x\in\mathbb{Z}_+$	q/p	q/p^2	$(q=1-p)$
		$f(y)=p\,q^{y-1}$	$y\in\{1,...\}$	$1/p$	q/p^2	$(y=x+1)$
HyperGeo.	$\mathsf{HG}(n,A,B)$	$f(x)=\frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$	$x\in 0,\cdots,n$	$n\,P$	$n\,P\,(1-P)\frac{N-n}{N-1}$	$(P=\frac{A}{A+B})$
Logistic	$\mathsf{Lo}(\mu,\beta)$	$f(x)=\frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$	$x\in\mathbb{R}$	μ	$\pi^2\beta^2/3$	
Log Normal	$\mathsf{LN}(\mu,\sigma^2)$	$f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}e^{-(\log x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}_+$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$	
Neg. Binom.	$\mathsf{NB}(\alpha,p)$	$f(x)=\binom{x+\alpha-1}{x}p^\alpha\,q^x$	$x\in\mathbb{Z}_+$	$\alpha q/p$	$\alpha q/p^2$	$(q=1-p)$
		$f(y)=\binom{y-1}{y-\alpha}p^\alpha\,q^{y-\alpha}$	$y\in\{\alpha,...\}$	α/p	$\alpha q/p^2$	$(y=x+\alpha)$
Normal	$\mathsf{No}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}$	μ	σ^2	
Pareto	$\mathsf{Pa}(\alpha,\epsilon)$	$f(x)=\alpha\,\epsilon^\alpha/x^{\alpha+1}$	$x\in(\epsilon,\infty)$	$\frac{\epsilon\,\alpha}{\alpha-1}$	$\frac{\epsilon^2\alpha}{(\alpha-1)^2(\alpha-2)}$	
Poisson	$\mathsf{Po}(\lambda)$	$f(x)=\frac{\lambda^x}{x!}e^{-\lambda}$	$x\in\mathbb{Z}_+$	λ	λ	
Snedecor F	$F(\nu_1,\nu_2)$	$f(x)=\frac{\Gamma(\frac{\nu_1+\nu_2}{2})(\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})}\times$ $x^{\frac{\nu_1-2}{2}}\left[1+\frac{\nu_1}{\nu_2}x\right]^{-\frac{\nu_1+\nu_2}{2}}$	$x\in\mathbb{R}_+$	$\frac{\nu_2}{\nu_2-2}$	$\left(\frac{\nu_2}{\nu_2-2}\right)^2\frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$	
Student t	t_ν	$f(x)=\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}}[1+x^2/\nu]^{-(\nu+1)/2}$	$x\in\mathbb{R}$	0	$\nu/(\nu-2)$	
Uniform	$\mathsf{Un}(a,b)$	$f(x)=\frac{1}{b-a}$	$x\in(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
Weibull	$\mathsf{We}(\alpha,\beta)$	$f(x)=\alpha\beta\,x^{\alpha-1}\,e^{-\beta\,x^\alpha}$	$x\in\mathbb{R}_+$	$\frac{\Gamma(1+\alpha^{-1})}{\beta^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)}{\beta^{2/\alpha}}$	