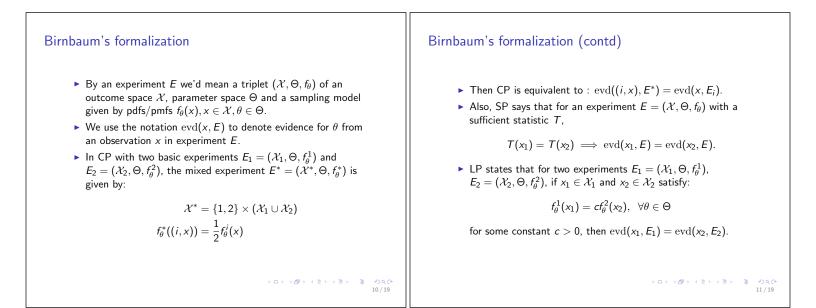
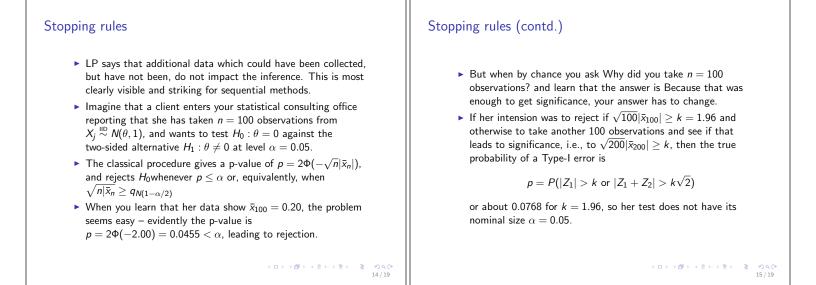


An example (contd.)	Example 2
 LP is violated here is due to the fact that p-value is the probability under H₀ of observing evidence against H₀ that is more extreme than the one in the recorded data. Such calculations clearly care about other possible data than what has been currently observed. This is common to all classical methods and it is well documented the concern about data that have not been observed can lead to absurd inference based on data that has indeed been observed! 	 Suppose X₁ and X₂ are independent with P(X_j = θ ± 1) = 1/2 for some unknown θ ∈ ℝ. The smallest 75% confidence interval for θ is C(X₁, X₂) = { the point X₁+X₂/2 if X₁ ≠ X₂/the point X₁ − 1 if X₁ ≠ X₂ so, P_θ(θ ∈ C(X₁, X₂)) = 0.75 for all θ. But once we observe X₁ and X₂, it is silly to report a 75% confidence. Instead we should report a confidence of 100% if X₁ ≠ X₂. ≈50% if X₁ = X₂. The problem here lies in not conditioning the inference on the observed data – again a violation of LP.
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Example 3 (Cox paradox)	Birnbaum's theorem
 A laboratory has two instruments for performing the same task, one has accuracy ±0.01 while the other has accuracy ±0.05. What accuracy should a scientist who gets to use whichever instrument is available (w.p. 1/2)? The one that she used or the average accuracy? 	 Birnbaum (1962) proved that LP is equivalent to the following two principles (CP) Conditionality principle. Suppose there are two experiments E₁ and E₂ where the only unknown is the parameter θ, common to the two problems. Consider the mixed experiment E_* in which we select i = 1 or i = 2 with equal probabilities, then perform experiment E_i; then the resulting evidence about θ is that from experiment E_i, and we can ignore the existence of the other (unperformed) experiment. (SP) Sufficiency principle. Consider an experiment E and a sufficient statistic T. Then if T(x₁) = T(x₂), the evidence about θ from observing x₂. Birnbaum showed LP ⇐→ CP + SP.
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$Proof \text{ of } CP + SP \implies LP$	Proof of CP + SP \implies LP (contd.)
 Suppose x₁ ∈ X₁, x₂ ∈ X₂ satisfy the LP condition for some c > 0. Define a statistic T : X* → X* as T((i,x)) = { (1,x₁) if i = 2, x = x₂ (i,x) otherwise Let X* ~ f_θ[*]. We'll show that the distribution of X* given T(X*) is free of θ. Indeed, if T(X*) = (1,x₁) then X* must equal T(X*) w.p. 1. if T(X*) = (1,x₁) then X* is either (1,x₁) or (2,x₂) with probabilities proportional to ½f_θ¹(x₁) and ½f_θ²(x₂), i.e., with probabilities c+1 and 1/c+1. 	 ► Therefore, because T((1, x₁)) = T((2, x₂)), evd(x₁, E₁) = evd((1, x₁), E*) [by CP] = evd((2, x₂), E*) [by SP] = evd(x₂, E₂) [by CP] as desired!
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Stopping rules (contd.)	Formalizing stopping rules
 To achieve this size she would have to reject when either √100 x 100 x 100 x 100 x 200 x 200 x 200 exceeds k = 2.12. Since hers do not, we now must change our advice and say she cannot reject H0! It is (or should be!) disturbing that the evidential import of her results should depend on her intentions, and not on the data and experiment. Even more alarming, most experiments are begun without a clear picture of when to stop taking data, so this silly example is in fact the usual situation. 	 Consider an infinite sequence of experiments E_m = (X_m, Θ, f^m_θ), m = 1, 2, ···. A stopping rule is a sequence of functions τ_m : X₁ × ··· × X_m → [0, 1] with the interpretation that we conduct the experiments sequentially, gathering data x₁ ∈ X₁, x₂ ∈ X₂, ··· and deciding at every step m whether to stop with probability τ_m(x₁, ··· , x_m) or otherwise to continue to the next step. A stopping rule is proper if it stops almost surely.
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The Stopping Rule Principle

• If τ is proper, the the sequential experiments can be put together to define the stopping-rule experiment $E^{(\tau)} = (\mathcal{X}^{(\tau)}, \Theta, f_{\theta}^{(\tau)})$ where

$$\begin{aligned} \mathcal{X}^{(\tau)} &= \{ (m, x_1, x_2, \cdots, x_m) : m \in \mathbb{N}, x_i \in \mathcal{X}_i \} \\ f_{\theta}^{(\tau)}((m, x_1, \cdots, x_m)) &= \tau_m(x_{1:m}) \left\{ \prod_{i=1}^{m-1} (1 - \tau_i(x_{1:i})) \right\} \prod_{i=1}^m f_{\theta}^i(x_i) \end{aligned}$$

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SRP (contd.)

• On the other hand, if we had decided beforehand to continue up to a fixed step *m*, then the corresponding *m*-step experiment is $E^{(m)} = (\mathcal{X}^{(m)}, \Theta, f_{\theta}^{(m)})$ where

$$\mathcal{X}^{(m)} = \{(x_1, x_2, \cdots, x_m) : x_i \in \mathcal{X}_i\}$$
$$f_{\theta}^{(m)}((x_1, \cdots, x_m)) = \prod_{i=1}^m f_{\theta}^i(x_i)$$

► The SRP states

$$\operatorname{evd}((m, x_1, \cdots, x_m), E^{(\tau)}) = \operatorname{evd}((x_1, \cdots, x_m), E^{(m)}).$$

► That is, once you stop at *m*, you can do inference pretending that you always wanted to do an *m*-step experiment.

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