Possible 961 Projects

Here are a few suggestions of possible topics for a Stochastic Processes term project; of course you're free (in fact encouraged) to modify the suggestions completely, or to pick something not on the list, from your own interests, other courses, *etc.*; this is only intended to help get you started in thinking about a possible project. Please ask me for more details, or more suggestions, or how to get started, or references, *etc.*

Most of these can be either *theoretical* projects, in which you might derive and prove the asymptotic results using martingales or difference or differential equations, or can be *simulation* exercises, in which you program a computer to run thousands of simulations of the life-history of rabbit and fox populations, epidemics, etc.

1. The classical **Predator-Prey** equations of Volterra and Lotka are:

$$\begin{aligned} \frac{dx}{dt} &= (A - By)x \\ \frac{dy}{dt} &= (Cx - D)y \end{aligned} \tag{VL}$$

   where $y(t)$ and $x(t)$ are the numbers of predators and prey, respectively (cats and mice, foxes and rabbits, ...); a good presentation appears on pages 258ff. of Hirsch & Smale's *Differential Equations, Dynamical Systems, and Linear Algebra*. Starting at an initial $x(0) = u > 0$ and $y(0) = v > 0$, this system evolves cyclically in time, with both population sizes rising and falling, returning again and again to (u, v) once each cycle.

   Propose and explore a *stochastic* predator-prey system, using a two-dimensional Markov chain (either continuous or discrete time, continuous- or integer-valued) describing the evolution of the two population sizes, but intended to be analogous to the ODE (VL). What is the asymptotic behavior? Contrast it with that of the deterministic system.

2. In the classical **Reed-Frost** model for epidemics, the population (of total size N) is taken to include "Susceptibles", "Infectives", and "Removed" (the latter group would include either or both of recovered and now-immune individuals and individuals who do not survive the illness). The *deterministic* discrete-time form of the evolution equations for the numbers s,i,r in these three groups is:

$$\begin{aligned} s_{t+1} &= s_t \cdot (q^{i_t}) \\ i_{t+1} &= s_t \cdot (1 - q^{i_t}) \\ r_{t+1} &= N - i_{t+1} - s_{t+1}, \end{aligned} \tag{RF}$$

   where p is the probability that any given infective and susceptible individual will meet during a time-period, and $q = 1 - p$. Show that the stochastic version (with random numbers I_t, S_t, R_t of individuals) has different limiting behavior from the