Outline of Stochastic Processes

I. Introduction and Course Objectives
  A. Unusual applications:
    1. Nonparametric Bayesian Analysis
      a. Density Estimation
      b. Regression
      c. Survival Analysis
    2. Stochastic Numerical Methods
      a. MCMC Integration (Gibbs, Metrop)
      b. Global Optimization (SA, GA)
      c. Convergence rates, diagnostics
  B. Inference
    1. Detecting & estimating drift
    2. Stationarity, Ergodicity

II. Review of Probability Theory
  A. Example: Infinite Coin Toss
    1. Event: "At least one tail by nth toss"
    2. Event: "At least one tail"
    3. Event: "Infinitely-many tails"
  B. Outcomes, Events, Random Variables
    1. Probability Assignments (Measures)
    2. Probability Spaces
  C. Distribution of Random Variables
    1. Canonical Spaces
    2. Expectation, Moments, ChF
    3. Independence
  D. Filtrations: Nested Sigma Algebras
    1. Conditional Expectation
      a. Regular Conditional Prob Dist'ns
      b. Independence
    2. Martingales, Supermartingales
      a. 'Conditionally Constant'
      b. MG Convergence Theorem
    3. Markov Times
      a. Hitting times
      b. Doob's Optional Sampling Theorem

III. Discrete State Space Processes
  A. Finite-state Markov Chains
    1. Review of relevent matrix theory
    2. Limiting behaviour, Aper Irreduc
    3. Ergodicity
    4. Recurrence and Renewal
    5. Metropolis/Hastings Algorithm
    6. Associated Martingales
      a. Generators for Markov Chain
      b. Equivalence of MGP and MC
  B. Random Walks
    1. Gambler's Ruin (1-dim)
      a. Difference Equations
      b. Martingale Methods
    2. SSRW in d dimensions
    3. Birth/Death
  C. Non-Markov processes

IV. Continuous State Space, Discrete Time Processes
  A. Markov Chains & Random Walks
    1. Ergodicity, Harris Recurrence
    2. MCMC: Gibbs
    3. MCMC: Metropolis
  B. Sequential Statistical Procedures
    1. SPRT
    2. Wald's Correction, Bayesian Versions
  C. Martingales
    1. Convergence Theorem
    2. Maximal Theorems
      a. Probability
      b. L^p
    3. Path properties
V. Discrete State Space, Continuous Time Processes
  A. Poisson Process
  B. Birth/Death Processes
  C. Continuous-Time Markov Chains
    1. Simulation Methods
    2. Limiting Behaviour
VI. Continuous State Space, Continuous Time Processes
  A. Generalized Poisson Process
    1. Stationary Independent-Increment Processes
    2. Stable Processes
    3. General Markov Processes
  B. Brownian Motion
    1. Brownian Bridge
    2. Limit of Random Walks
    3. Limit of Poisson Birth/Death Processes
    4. Nonparametric Bayes
      a. Modelling log-PDF's
      b. Modelling Regression Functions
  C. Semimartingales
    1. Path Properties
    2. Diffusions, Genertors
  D. Gaussian Processes
    1. Karhunen-Loève Expansion
    2. Covariance Functionals
    3. Reproducing Kernel Hilbert Spaces
  E. Multiparameter Processes, Random Fields
    1. Generalized Processes
    2. Kriging and Spatial Statistics
    3. Poisson Point Processes
      a. Modelling Biodiversity