Midterm Examination # 2

Mth 136 = Sta 114

Thursday, 2011 April 14
2:50 – 4:05 pm

- This is a **closed book** exam— please put your books on the floor.
- You may use a calculator and **one page** of your own notes.
- Do not share calculators or notes. No cell phones, please.
- **Show** your **work**. Neatness counts. Boxing answers helps.
- Numerical answers: **four significant digits** or fractions **in lowest terms**. Simplify *all* answers.
- Blank worksheet and pdf & distribution tables are attached.

Cheating on exams is a breach of trust with classmates and faculty that will not be tolerated. After completing the exam please acknowledge that you have adhered to the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

DCS Affirmation:

Signature: ______________________

Print Name: ______________________

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
Total:	/100

Problem 1: Let $X \sim \mathsf{Un}(0, \theta)$ be a single draw from the uniform distribution on the interval $[0, \theta]$ for some uncertain $\theta > 0$. Consider the problem of estimating $g(\theta) = \theta^2$ with an estimator of the form

$$\delta(X) = aX^2$$

for some constant $a \in \mathbb{R}$.

a) Find the mean and the variance of the estimator $\delta(X) = aX^2$:
$\mathsf{E}_\theta[\delta(X)] =$ $\mathsf{Var}_\theta[\delta(X)] =$

b) For what value of a (if any) is $\delta(X) = aX^2$ the Maximum Likelihood Estimator of θ^2?
$a =$

Problem 1 (cont):

c) For what value of a (if any) is $\delta(X) = aX^2$ an Unbiased Estimator of θ^2?

$a =$

d) For what value of $a \in \mathbb{R}$ does $\delta(X) = aX^2$ have the smallest Mean Square Error for estimating θ^2?

$a =$

XC: Bored? For +2pts, answer a) – d) for $\delta(x) = a \max_{1\le i\le n}(X_i^2)$ for a sample of size n.

Problem 2: The independent count data $\mathbf{x} = \{X_1, \cdots, X_n\} \overset{\text{iid}}{\sim} \mathsf{Ge}(p)$ come from the geometric distribution with pmf

$$\mathsf{P}[X_i = k] = p\, q^k, \qquad k = 0, 1, 2, \dots$$

with uncertain parameter $p \in (0, 1)$, where $q \equiv (1 - p)$. Let $S_n = \sum_{i \le n} X_i$ be the sum of the n observations. We wish to test the hypotheses:

$$H_0: \; p = 0.50 \qquad vs. \qquad H_1: \; p = 0.25$$

a) Consider a test based on just $n = 1$ observation X_1, that rejects H_0 in favor of H_1 when $X_1 \in \mathcal{R} = \{4, 5, 6, 7, 8, 9, ...\}$. Give the exact numerical size α and power $1 - \beta$ of this test: (Simplify!)

$\alpha =$ $\qquad\qquad$ $1 - \beta =$

b) Find the Bayesian posterior probability of H_0, with equal prior probabilities of $\pi_0 = \pi_1 = 1/2$ for H_0 and H_1, for a single observation $X_1 = x$. Simplify!

$\mathsf{P}[H_0 \mid X_1 = x] =$

Problem 2 (cont):

As before, we have $\mathbf{x} = \{X_1, \cdots, X_n\} \overset{\text{iid}}{\sim} \mathsf{Ge}(p)$ and wish to test

$$H_0: \ p = 0.50 \qquad vs. \qquad H_1: \ p = 0.25$$

c) If H_0 is true, what is the *exact* probability distribution of $S_n = \sum_{i=1}^n X_i$? Give its name and the value(s) of any parameter(s). Also, what is its *approximate* normal distribution (by the CLT), if n is large?

$S_n \sim$ $\approx \mathsf{No}\Big(\mu =$ ________, $\sigma^2 =$ ________$\Big)$

d) Find the (approximate[1]) P-value for a test of H_0 *vs.* H_1 for a sample of size $n = 200$ with $S_{200} = 250$. Do you Reject H_0 or not at level $\alpha = 0.01$?

$P \approx$ Reject? Y N

[1]Part c) should be helpful

Problem 3: The count data $\mathbf{x} = \{X_1, \cdots, X_n\}$ come from either the Poisson distribution or the Geometric distribution, but we're not sure which. We do know that the mean is $\mu = 5$. Consider the two hypotheses

$$H_0: \ \{X_i\} \overset{\text{iid}}{\sim} \mathsf{Po}(5) \qquad vs. \qquad H_1: \{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ge}(1/6)$$

and the four statistics:

$$S_n(\mathbf{x}) = \sum_{i\le n} X_i \qquad T_n(\mathbf{x}) = \sum_{i\le n} X_i^2$$
$$U_n(\mathbf{x}) = \sum_{i\le n} \log(X_i) \qquad V_n(\mathbf{x}) = \sum_{i\le n} \log(X_i!)$$

a) Under both H_0 and H_1 the data have mean $\mathsf{E}[X_i] = 5$, but the variances are different— the $\mathsf{Po}(\theta)$ variance is θ, or 5 for $\theta = 5$, while the $\mathsf{Ge}(p)$ variance is much bigger— q/p^2, or 30 for $p = 1/6$. Thus the statistic

$$W_n(\mathbf{x}) = \sum_{i\le n} (X_i - 5)^2 = T_n(\mathbf{x}) - 10S_n(\mathbf{x}) + 25n$$

ought to be bigger if H_1 is true than if H_0 is true. Find:[2]

$\mathsf{E}[W_n \mid H_0] =$ $\qquad\qquad\qquad$ $\mathsf{E}[W_n \mid H_1] =$

[2]Hint: This is easy; don't do something hard!

Problem 3 (cont):

b) Express the logarithm of the likelihood ratio statistic (against H_0)

$$\Lambda_n(\mathbf{x}) = \frac{f_1(\mathbf{x})}{f_0(\mathbf{x})}$$

for a sample of size n, in terms of one or more of $\{S_n, T_n, U_n, V_n\}$. Simplify!
$\log \Lambda_n(\mathbf{x}) =$

c) Two possible tests of H_0 *vs.* H_1 are:

(1): Reject H_0 if $[W_n(\mathbf{x}) \geq c_1]$ (2): Reject H_0 if $[\log \Lambda_n(\mathbf{x}) \geq c_2]$

for specified critical values c_1 and c_2. Denote the size and power of Test (1) (based on W_n) by α_1 and $1-\beta_1$, and those of Test (2) (based on $\log \Lambda_n$) by α_2 and $1-\beta_2$, respectively (these will depend on c_1, c_2, and n).
Which is test is better, test (1) or (2) (circle one) and, in 25 words or less, in terms of the α's and β's, *why*? Be specific.

Problem 4: Two brands of floor tile are being compared for their scratch resistance. A number m of measurements $\{X_1, \cdots, X_m\}$ are taken for AjaX tile; another set of n measurements $\{Y_1, \cdots, Y_n\}$ are taken for BiffY tile. All the measurements are independent and normally distributed, with the same variance σ^2, but perhaps the mean μ_x of the AjaX tiles is different from that μ_y of BiffY Tile. Thus we wish to test:

$$H_0: \ \mu_x = \mu_y \quad vs. \quad H_1: \ \mu_x \neq \mu_y$$

with $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu_x, \sigma^2)$, $\{Y_j\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu_y, \sigma^2)$.
Here are statistics from some data, with sample-sizes $m = 10$ and $n = 12$:

$$\begin{aligned} \bar{x}_m &= \tfrac{1}{m}\sum x_i &&= 10.0 & \bar{y}_n &= \tfrac{1}{n}\sum y_j &&= 13.5 \\ SS_x &= \sum (x_i - \bar{x}_m)^2 &&= 100.0 & SS_y &= \sum (y_j - \bar{y}_n)^2 &&= 220.0 \end{aligned}$$

a) If σ^2 is unknown, find the (approximate) P-value for H_0:
$P \approx$

b) Find a 90% confidence interval for $\mu_x - \mu_y$:

Problem 4 (cont):

c) Do you *Reject* H_0 at level $\alpha = 0.10$? Y N (circle one)
Explain (in 25 words or less) how you could use *each* of your answers to parts a) and to part b) above to answer this (give both explanations).

d) Find a 90% confidence interval for $\mu_x - \mu_y$ if we learn that $\sigma^2 = 9.0$ exactly. Did you use a Normal distribution or a Student's t distribution for this? If t, how many degrees of freedom? If Normal, why?

Problem 5: For 2pt each, write your answers in the boxes provided, or circle True or False. No explanations are required. Parameterizations and pdfs for distributions ($\mathsf{Be}, \mathsf{Bi}, \mathsf{Ex}, \mathsf{Ga}, \mathsf{Ge}, \mathsf{No}, \mathsf{Po}, \ldots$) are on Page 13.

a) The "P-value" is the probability that H_0 is true. T F

b) If $X \sim \mathsf{Ex}(\lambda)$, what is the P-value for $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$ for one observation $X = \log 3 \approx 1.098612$? [ ]

c) In regular problems, no unbiased estimator of θ can have MSE below $\frac{1}{nI(\theta)}$, where $I(\theta)$ is the Fisher information: T F

d) If a $\gamma = 95\%$ Confidence Interval for θ does *not* contain θ_0, we can Reject the hypothesis $H_0 :\ \theta = \theta_0$ at level $\alpha = 0.05$: T F

e) When testing $H_0 :\ \mu = \mu_0$ for $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu, \sigma^2)$, if $\bar{X}_n > \mu_0$ then you should use $H_1 :\ \mu > \mu_0$ for the alternate hypothesis. T F

f) If X, Y are independent then $\mathsf{Var}[X - Y] = \mathsf{Var}[X] - \mathsf{Var}[Y]$: T F

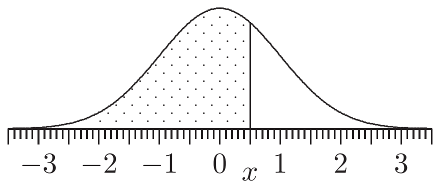
g) For $\mathbf{x} = \{X_i\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu, \sigma^2)$, there does not exist a Uniformly Most Powerful test of $H_0 :\ \mu = 0$ *vs.* $H_1 :\ \mu \neq 0$ T F

h) For $\mathbf{x} = \{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ex}(\lambda)$, there does not exist a Uniformly Most Powerful test of $H_0 :\ \lambda = 1$ *vs.* $H_1 :\ \lambda > 1$ T F

i) If $X \sim \mathsf{Ex}(\lambda)$, what is the posterior probability of $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$ for one obs'n $X = \log 3$, with equal prior probabilities? [ ]

j) If X and Y each have Student t_2 distributions and are independent, then $X + Y$ has a t_4 distribution. T F

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz:$$



Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

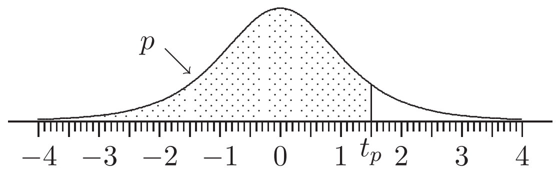
x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$

$\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$

Critical Values for Student's t

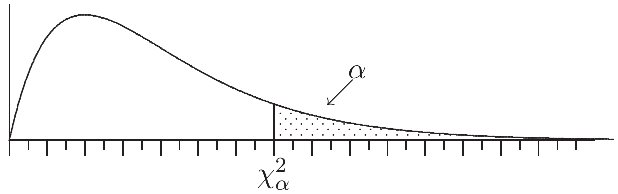
$$p = \int_{-\infty}^{t_p} c \, \frac{dt}{(1 + t^2/\nu)^{(\nu+1)/2}}$$



ν	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$	$t_{.9999}$
1	0.325	0.727	1.376	1.9626	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.3862	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.2498	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.1896	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.1558	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.1342	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.1192	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.1081	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.0997	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.0931	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.0877	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.0832	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.0795	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.0763	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.0735	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.0711	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.0690	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.0672	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.0655	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.0640	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.0627	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.0614	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.0603	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.0593	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.0584	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.0575	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.0567	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.0560	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.0553	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.0547	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.0500	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.0455	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.0409	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.0364	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

Critical Values for χ^2

$$\alpha = \int_{\chi^2_\alpha}^{\infty} c\, x^{\nu/2-1} e^{-x/2}\, dx$$



ν	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	$\chi^2_{.001}$	$\chi^2_{.0005}$	$\chi^2_{.0001}$
1	0.4549	1.3233	2.7055	3.8415	5.0239	6.6349	7.87940	10.8276	12.1157	15.1367
2	1.3863	2.7726	4.6052	5.9915	7.3778	9.2103	10.5966	13.8155	15.2018	18.4207
3	2.3660	4.1083	6.2514	7.8147	9.3484	11.3449	12.8382	16.2662	17.7300	21.1075
4	3.3567	5.3853	7.7794	9.4877	11.1433	13.2767	14.8603	18.4668	19.9974	23.5127
5	4.3515	6.6257	9.2364	11.0705	12.8325	15.0863	16.7496	20.5150	22.1053	25.7448
6	5.3481	7.8408	10.6446	12.5916	14.4494	16.8119	18.5476	22.4577	24.1028	27.8563
7	6.3458	9.0371	12.0170	14.0671	16.0128	18.4753	20.2777	24.3219	26.0178	29.8775
8	7.3441	10.219	13.3616	15.5073	17.5345	20.0902	21.9550	26.1245	27.8680	31.8276
9	8.3428	11.389	14.6837	16.9190	19.0228	21.6660	23.5894	27.8772	29.6658	33.7199
10	9.3418	12.549	15.9872	18.3070	20.4831	23.2092	25.1882	29.5883	31.4198	35.5640
11	10.341	13.701	17.2750	19.6751	21.9200	24.7249	26.7568	31.2641	33.1366	37.3670
12	11.340	14.845	18.5493	21.0260	23.3366	26.2169	28.2995	32.9095	34.8213	39.1344
13	12.340	15.984	19.8119	22.3620	24.7356	27.6882	29.8195	34.5282	36.4778	40.8707
14	13.339	17.117	21.0641	23.6848	26.1189	29.1412	31.3193	36.1233	38.1094	42.5793
15	14.339	18.245	22.3071	24.9958	27.4884	30.5779	32.8013	37.6973	39.7188	44.2632
16	15.338	19.369	23.5418	26.2962	28.8453	31.9999	34.2672	39.2524	41.3081	45.9249
17	16.338	20.489	24.7690	27.5871	30.1910	33.4087	35.7185	40.7902	42.8792	47.5664
18	17.338	21.605	25.9894	28.8693	31.5264	34.8053	37.1565	42.3124	44.4338	49.1894
19	18.338	22.718	27.2036	30.1435	32.8523	36.1909	38.5823	43.8202	45.9731	50.7955
20	19.337	23.828	28.4120	31.4104	34.1696	37.5662	39.9968	45.3147	47.4985	52.3860
21	20.337	24.935	29.6151	32.6706	35.4789	38.9322	41.4011	46.7970	49.0108	53.9620
22	21.337	26.039	30.8133	33.9244	36.7807	40.2894	42.7957	48.2679	50.5111	55.5246
23	22.337	27.141	32.0069	35.1725	38.0756	41.6384	44.1813	49.7282	52.0002	57.0746
24	23.337	28.241	33.1962	36.4150	39.3641	42.9798	45.5585	51.1786	53.4788	58.6130
25	24.337	29.339	34.3816	37.6525	40.6465	44.3141	46.9279	52.6197	54.9475	60.1403
26	25.336	30.435	35.5632	38.8851	41.9232	45.6417	48.2899	54.0520	56.4069	61.6573
27	26.336	31.528	36.7412	40.1133	43.1945	46.9629	49.6449	55.4760	57.8576	63.1645
28	27.336	32.620	37.9159	41.3371	44.4608	48.2782	50.9934	56.8923	59.3000	64.6624
29	28.336	33.711	39.0875	42.5570	45.7223	49.5879	52.3356	58.3012	60.7346	66.1517
30	29.336	34.800	40.2560	43.7730	46.9792	50.8922	53.6720	59.7031	62.1619	67.6326
40	39.336	45.616	51.8051	55.7585	59.3417	63.6907	66.7660	73.4020	76.0946	82.0623
50	49.335	56.334	63.1671	67.5048	71.4202	76.1539	79.4900	86.6608	89.5605	95.9687
60	59.335	66.981	74.3970	79.0819	83.2977	88.3794	91.9517	99.6072	102.695	109.503
70	69.335	77.577	85.5270	90.5312	95.0232	100.425	104.215	112.317	115.578	122.755
80	79.334	88.130	96.5782	101.879	106.629	112.329	116.321	124.839	128.261	135.782
90	89.334	98.650	107.565	113.145	118.136	124.116	128.299	137.208	140.782	148.627
100	99.334	109.14	118.498	124.342	129.561	135.807	140.169	149.449	153.167	161.319

Name	Notation	pdf/pmf	Range	Mean μ	Variance σ^2	
Beta	$\mathsf{Be}(\alpha,\beta)$	$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$x\in(0,1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Binomial	$\mathsf{Bi}(n,p)$	$f(x)=\binom{n}{x}p^x q^{(n-x)}$	$x\in 0,\cdots,n$	$n\,p$	$n\,p\,q$	$(q=1-p)$
Exponential	$\mathsf{Ex}(\lambda)$	$f(x)=\lambda\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$	
Gamma	$\mathsf{Ga}(\alpha,\lambda)$	$f(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	α/λ	α/λ^2	
Geometric	$\mathsf{Ge}(p)$	$f(x)=p\,q^x$	$x\in\mathbb{Z}_+$	q/p	q/p^2	$(q=1-p)$
		$f(y)=p\,q^{y-1}$	$y\in\{1,...\}$	$1/p$	q/p^2	$(y=x+1)$
HyperGeo.	$\mathsf{HG}(n,A,B)$	$f(x)=\frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$	$x\in 0,\cdots,n$	$n\,P$	$n\,P\,(1-P)\frac{N-n}{N-1}$	$(P=\frac{A}{A+B})$
Logistic	$\mathsf{Lo}(\mu,\beta)$	$f(x)=\frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$	$x\in\mathbb{R}$	μ	$\pi^2\beta^2/3$	
Log Normal	$\mathsf{LN}(\mu,\sigma^2)$	$f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}e^{-(\log x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}_+$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$	
Neg. Binom.	$\mathsf{NB}(\alpha,p)$	$f(x)=\binom{x+\alpha-1}{x}p^\alpha\,q^x$	$x\in\mathbb{Z}_+$	$\alpha q/p$	$\alpha q/p^2$	$(q=1-p)$
		$f(y)=\binom{y-1}{y-\alpha}p^\alpha\,q^{y-\alpha}$	$y\in\{\alpha,...\}$	α/p	$\alpha q/p^2$	$(y=x+\alpha)$
Normal	$\mathsf{No}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}$	μ	σ^2	
Pareto	$\mathsf{Pa}(\alpha,\epsilon)$	$f(x)=\alpha\,\epsilon^\alpha/x^{\alpha+1}$	$x\in(\epsilon,\infty)$	$\frac{\epsilon\,\alpha}{\alpha-1}$	$\frac{\epsilon^2\alpha}{(\alpha-1)^2(\alpha-2)}$	
Poisson	$\mathsf{Po}(\lambda)$	$f(x)=\frac{\lambda^x}{x!}e^{-\lambda}$	$x\in\mathbb{Z}_+$	λ	λ	
Snedecor F	$F(\nu_1,\nu_2)$	$f(x)=\frac{\Gamma(\frac{\nu_1+\nu_2}{2})(\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})}\times$ $x^{\frac{\nu_1-2}{2}}\left[1+\frac{\nu_1}{\nu_2}x\right]^{-\frac{\nu_1+\nu_2}{2}}$	$x\in\mathbb{R}_+$	$\frac{\nu_2}{\nu_2-2}$	$\left(\frac{\nu_2}{\nu_2-2}\right)^2\frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$	
Student t	$t(\nu)$	$f(x)=\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}}[1+x^2/\nu]^{-(\nu+1)/2}$	$x\in\mathbb{R}$	0	$\nu/(\nu-2)$	
Uniform	$\mathsf{Un}(a,b)$	$f(x)=\frac{1}{b-a}$	$x\in(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
Weibull	$\mathsf{We}(\alpha,\beta)$	$f(x)=\alpha\beta\,x^{\alpha-1}\,e^{-\beta\,x^\alpha}$	$x\in\mathbb{R}_+$	$\frac{\Gamma(1+\alpha^{-1})}{\beta^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)}{\beta^{2/\alpha}}$	

Name: ____________

Mth 136 = Sta 114

(Nearly) Blank worksheet, if needed:

Another extra worksheet, if needed: