Midterm Examination I

STA 532: Statistical Inference

Thursday, 2016 Mar 3, 1:25 – 2:40pm

This is a closed-book exam. You may use a single sheet of prepared notes, if you wish, but you may not share materials.

If a question seems ambiguous or confusing, *please* ask me to clarify it. Unless a problem states otherwise, you must **show** your **work**. There are blank worksheets at the end of the test if you need more room for this.

It is to your advantage to write your solutions as clearly as possible, and to box answers I might not find.

For full credit, answers must be given in **closed form** with no unevaluated sums, integrals, maxima, unreduced fractions, *etc.* Wherever possible, **simplify**.

Good luck!

Print Name Clearly: ______________________________

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
Total:	/100

Problem 1: Let $\mathbf{x} = \{X_i\}_{1 \le i \le n}$ be a simple random sample of n independent random variables with the exponential $\mathsf{Ex}(\theta)$ distribution with rate θ, or pdf

$$f(x) = \theta e^{-\theta x}\,\mathbf{1}_{\{x>0\}}.$$

a) (5) Derive the maximum likelihood estimator $\hat{\theta}_n(\mathbf{x})$ of θ for a sample of size n:
$\hat{\theta}_n(\mathbf{x}) =$

b) (5) Find[1] the exact distribution of the statistic $T_n(\mathbf{x}) := \min_{1 \le i \le n} X_i$, the sample minimum, either by name with any parameter value(s), or by giving the pdf correctly for all t:
$T_n \sim$

[1]Hint: What's $\mathsf{P}[T_n(\mathbf{x}) > t]$ for $t > 0$?

Problem 1 (cont'd): Still $\mathbf{x} = \{X_i\}_{1\le i\le n} \overset{\text{iid}}{\sim} \mathsf{Ex}(\theta)$ and $T_n := \min_{1\le i\le n}\{X_i\}$.

c) (5) Find the maximum likelihood estimator $\hat{\theta}_T(t)$ of θ for one observation $T_n = t$ (Just think of T_n as a random variable whose distribution depends on θ):
$\hat{\theta}_T(t) =$

d) (5) Is T_n a sufficient statistic of $\mathbf{x}$ for θ? ◯ Yes ◯ No
If so, explain why; if not, find a real-valued statistic $S_n(\mathbf{x})$ that *is* sufficient:

Problem 2: Consider the simple random sample

$$\mathbf{x} = \{X_i\} = \{0,\ 2,\ 5,\ 3,\ 1,\ 9,\ 1,\ 5,\ 2,\ 6\}$$

of size n from the Poisson distribution $\mathsf{Po}(\theta)$ with mean θ, or pmf

$$f(x) = \frac{\theta^x}{x!} e^{-\theta}, \quad x = 0, 1, \ldots.$$

a) (6) Let θ have a Gamma prior distribution with mean $\mathsf{E}_\pi[\theta] = 1$ and variance $\mathsf{V}_\pi[\theta] = 1/2$. Find the posterior distribution for θ, upon observing the data above. Give either its pdf (correctly for all θ) or its name and the value(s) of any parameter(s).

$\pi(\theta \mid \mathbf{x}) \sim$

b) (4) Find both the MLE $\hat{\theta}(\mathbf{x})$ and the Bayesian estimate $\bar{\theta}_\pi(\mathbf{x})$ (using the prior from a) above) under squared-error loss:

$\hat{\theta}(\mathbf{x}) =$ ____________ $\bar{\theta}_\pi(\mathbf{x}) =$ ____________

Problem 2 (cont'd): Still $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Po}(\theta)$ and $\mathbf{x} = \{0, 2, 5, 3, 1, 9, 1, 5, 2, 6\}$.

c) (5) Use a normal approximation to the posterior distribution of θ that you found in a) above to find a 95% Credible Interval for θ, *i.e.*, find the end-points a, b of an interval such that $\mathsf{P}_\pi[a \le \theta \le b \mid \mathbf{x}] \approx 0.95$:

d) (5) Find the Jeffreys' prior distribution π_J by either specifying the pdf correctly at all $\theta \in \mathbb{R}$ or by giving the distribution name and parameter(s), and also the Jeffreys' *posterior* distribution for these data:

$\pi_J(\theta) \sim$ $\qquad\qquad$ $\pi_J(\theta \mid \mathbf{x}) \sim$

Problem 3: The number X_j of failures in a communication line has a Poisson distribution with mean θ, for $1 \le j \le n$, all independent; each line only works if it has zero failures. We would like to learn about θ from the data $\mathbf{x} = \{x_j\}$ but, unfortunately, all we can observe for each line is the indicator variable of whether or not the line works,

$$Y_j := \begin{cases} 1 & X_j = 0 \\ 0 & X_j \ge 1 \end{cases} \qquad 1 \le j \le n.$$

a) (2) Denote the expectation of Y_j by p, a function of θ. What is $p(\theta)$?
$p(\theta) := \mathsf{E}_\theta[Y_j] = \mathsf{P}_\theta[X_j = 0] =$

b) (4) Find a statistic $T_n(\mathbf{y})$ of the sample $\mathbf{y} = \{y_i\}_{1 \le i \le n}$ that is sufficient for θ and write the likelihood function for θ in terms of $T_n(\mathbf{y})$. Simplify!
$T_n(\mathbf{y}) =$ $\qquad\qquad$ $f_n(\mathbf{y} \mid \theta) =$

c) (4) What is the maximum likelihood estimate of θ, based only on the observations $\mathbf{y} = \{y_j\}_{1 \le j \le n}$?
$\hat{\theta}_n(\mathbf{y}) =$

Problem 3 (cont'd): Still $\{X_j\} \overset{\text{iid}}{\sim} \mathsf{Po}(\theta)$ and $Y_j := \mathbf{1}_{\{X_j=0\}}$.

d) (4) What is the MLE of $p = p(\theta)$, based on a sample $\mathbf{x} = \{x_j\}_{1\le j\le n}$ of the actual Poisson counts?

$\hat{p}_n(\mathbf{x}) =$

e) (4) $\bar{X}_n \to \theta$ as $n \to \infty$ by the Law of Large Numbers. Use the Delta Method to find the approximate mean and variance of $\hat{p}_n(\mathbf{x})$.

$\mathsf{E}_\theta[\hat{p}_n(\mathbf{x})] \approx$ $\qquad$ $\mathsf{Var}_\theta[\hat{p}_n(\mathbf{x})] \approx$

f) (2) Which do you expect to be a more efficient estimator of p:

◯ $\hat{p}_n(\mathbf{x})$, the MLE based on $\mathbf{x}$, or

◯ $\bar{Y}_n := \frac{1}{n}\sum_{1\le j\le n} Y_j$, the MLE based on $\mathbf{y}$? Why?

OPTIONAL **extra credit** for calculating the MSE of each and proving your choice of which is most efficient:

$\mathsf{E}_\theta|\hat{p}_n(\mathbf{x}) - p|^2 \approx$ $\qquad$ $\mathsf{E}_\theta|\bar{Y}_n - p|^2 \approx$

Problem 4: The lifetimes $\{X_i\}_{1\le i\le n} \overset{\text{iid}}{\sim} \mathsf{Pa}(\theta, 2)$ of blixtlepods (in minutes) have independent Pareto distributions, with pdf

$$f(x \mid \theta) = (\theta/2)\big/(1 + x/2)^{\theta+1}\,\mathbf{1}_{\{x>0\}}$$

and CDF $F(x \mid \theta) = \mathsf{P}_\theta[X_i \le x] = 1 - (1 + x/2)^{-\theta}$ for $x > 0$ for some uncertain parameter $\theta > 0$.

a) (5) Find[2] the Method of Moments (MoM) estimate of θ:
$\tilde{\theta}_n =$

b) (5) Find the log likelihood function for these data:
$\ell_{\mathbf{x}}(\theta) = \log f_n(\mathbf{x} \mid \theta) =$

[2]Recall the sheet of common distributions' features on page 13 of this exam.

Problem 4 (cont'd): Still $\{X_i\}_{1\le i\le n} \stackrel{\text{iid}}{\sim} \mathsf{Pa}(\theta, 2)$ have pdf

$$f(x \mid \theta) := (\theta/2)\big/(1 + x/2)^{\theta+1}\, \mathbf{1}_{\{x>0\}}$$

c) (5) Find the Fisher Information for a single observation from this distribution:

$I(\theta) =$

d) (5) What is the posterior distribution for this sample of size n, for the Jeffreys' Rule prior? Give its pdf or name & parameter value(s). Simplify!

$\pi_J(\theta \mid \mathbf{x}) \sim$

Problem 5: For 2pt each, write your answers in the boxes provided, or circle True or False. No explanations are required. Parameterizations and pdfs for distributions ($\mathsf{Be}, \mathsf{Bi}, \mathsf{Ex}, \mathsf{Ga}, \mathsf{Ge}, \mathsf{No}, \mathsf{Po}, \cdots$) are on Page 13.

a) If $X \sim \mathsf{Ga}(\alpha, \lambda)$ then $\mathsf{E}[1/X] = \lambda/\alpha$. T F

b) If $X \sim \mathsf{No}(0, 1)$ and $Y \sim \mathsf{Ga}(\theta, \theta)$ are independent then $Z := X/\sqrt{Y}$ has a t distribution. T F (If true, deg fdm $\nu =$) [ ]

c) The posterior mean $T(\mathbf{x}) = \mathsf{E}[\theta \mid \mathbf{x}]$ minimizes $\mathsf{E}\big[|T(\mathbf{x}) - \theta| \ \mid \mathbf{x}\big]$. T F

d) If $\sum_{i \le 9} X_i = 63$ for $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu, 1)$, then $\mathsf{P}_\mu[7 - 1.96/3 \le \mu \le 7 + 1.96/3] \approx 0.95$. T F

e) The MoM estimator of λ from $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ga}(3, \lambda)$ is $\tilde{\lambda} =$ [ ]

f) If $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ga}(\alpha, 2)$ then $T(\mathbf{x}) := \prod X_i$ is sufficient for α. T F

g) The smallest possible MSE for an unbiased estimator T of the mean μ of $\{X_i\}_{1 \le i \le 4} \overset{\text{iid}}{\sim} \mathsf{No}(\mu, 9)$ is: $\mathsf{E}[|T(\mathbf{x}) - \mu|^2] \ge$ [ ]

h) The mean is undefined for the Cauchy dist'n with pdf $f(x) = \frac{1/\pi}{1+(x-m)^2}$, because the tails are too "fat." T F

i) If $X \sim \mathsf{Bi}(6, \theta)$ and θ has a $\mathsf{Un}(0, 1)$ prior, the posterior mean of θ for single observation $\mathbf{x} = \{4\}$ is $\mathsf{E}[\theta \mid \mathbf{x}] =$ [ ]

j) If $\{X_n\} \overset{\text{iid}}{\sim} \mathsf{No}(0, 1)$, then $\sum_1^{10} (X_j)^2 \sim \mathsf{Ga}(\alpha, \lambda)$ for what values of α and λ? (1pt each) [ ]

Blank worksheet, if needed:

Normal Distribution Table

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz:$$

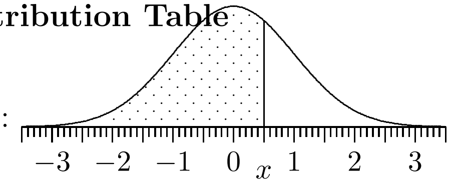


Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

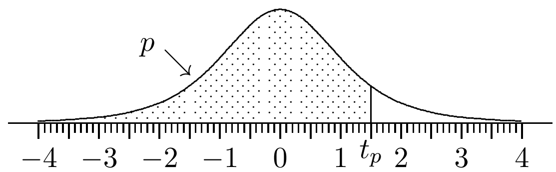
x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$

$\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$

Critical Values for Student's t

$$p = \int_{-\infty}^{t_p} c \, \frac{dt}{(1 + t^2/\nu)^{(\nu+1)/2}}$$



ν	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$	$t_{.9999}$
1	0.325	0.727	1.376	1.9626	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.3862	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.2498	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.1896	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.1558	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.1342	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.1192	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.1081	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.0997	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.0931	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.0877	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.0832	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.0795	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.0763	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.0735	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.0711	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.0690	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.0672	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.0655	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.0640	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.0627	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.0614	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.0603	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.0593	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.0584	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.0575	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.0567	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.0560	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.0553	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.0547	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.0500	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.0455	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.0409	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.0364	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

Name	Notation	pdf/pmf	Range	Mean μ	Variance σ^2	
Beta	$\mathsf{Be}(\alpha,\beta)$	$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$x\in(0,1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Binomial	$\mathsf{Bi}(n,p)$	$f(x)=\binom{n}{x}p^x q^{(n-x)}$	$x\in 0,\cdots,n$	$n\,p$	$n\,p\,q$	$(q=1-p)$
Exponential	$\mathsf{Ex}(\lambda)$	$f(x)=\lambda\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$	
Gamma	$\mathsf{Ga}(\alpha,\lambda)$	$f(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	α/λ	α/λ^2	
Geometric	$\mathsf{Ge}(p)$	$f(x)=p\,q^x$	$x\in\mathbb{Z}_+$	q/p	q/p^2	$(q=1-p)$
		$f(y)=p\,q^{y-1}$	$y\in\{1,...\}$	$1/p$	q/p^2	$(y=x+1)$
HyperGeo.	$\mathsf{HG}(n,A,B)$	$f(x)=\frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$	$x\in 0,\cdots,n$	$n\,P$	$n\,P\,(1-P)\frac{N-n}{N-1}$	$(P=\frac{A}{A+B})$
Logistic	$\mathsf{Lo}(\mu,\beta)$	$f(x)=\frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$	$x\in\mathbb{R}$	μ	$\pi^2\beta^2/3$	
Log Normal	$\mathsf{LN}(\mu,\sigma^2)$	$f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}e^{-(\log x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}_+$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$	
Neg. Binom.	$\mathsf{NB}(\alpha,p)$	$f(x)=\binom{x+\alpha-1}{x}p^\alpha\,q^x$	$x\in\mathbb{Z}_+$	$\alpha q/p$	$\alpha q/p^2$	$(q=1-p)$
		$f(y)=\binom{y-1}{y-\alpha}p^\alpha\,q^{y-\alpha}$	$y\in\{\alpha,...\}$	α/p	$\alpha q/p^2$	$(y=x+\alpha)$
Normal	$\mathsf{No}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}$	μ	σ^2	
Pareto	$\mathsf{Pa}(\alpha,\epsilon)$	$f(x)=(\alpha/\epsilon)(1+x/\epsilon)^{-\alpha-1}$	$x\in\mathbb{R}_+$	$\frac{\epsilon}{\alpha-1}{}^*$	$\frac{\epsilon^2\alpha}{(\alpha-1)^2(\alpha-2)}{}^*$	
		$f(y)=\alpha\,\epsilon^\alpha/y^{\alpha+1}$	$y\in(\epsilon,\infty)$	$\frac{\epsilon\,\alpha}{\alpha-1}{}^*$	$\frac{\epsilon^2\alpha}{(\alpha-1)^2(\alpha-2)}{}^*$	$(y=x+\epsilon)$
Poisson	$\mathsf{Po}(\lambda)$	$f(x)=\frac{\lambda^x}{x!}e^{-\lambda}$	$x\in\mathbb{Z}_+$	λ	λ	
Snedecor F	$F(\nu_1,\nu_2)$	$f(x)=\frac{\Gamma(\frac{\nu_1+\nu_2}{2})(\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})}\times$ $x^{\frac{\nu_1-2}{2}}\left[1+\frac{\nu_1}{\nu_2}x\right]^{-\frac{\nu_1+\nu_2}{2}}$	$x\in\mathbb{R}_+$	$\frac{\nu_2}{\nu_2-2}{}^*$	$\left(\frac{\nu_2}{\nu_2-2}\right)^2\frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}{}^*$	
Student t	$t(\nu)$	$f(x)=\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}}[1+x^2/\nu]^{-(\nu+1)/2}$	$x\in\mathbb{R}$	0^*	$\nu/(\nu-2)^*$	
Uniform	$\mathsf{Un}(a,b)$	$f(x)=\frac{1}{b-a}$	$x\in(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
Weibull	$\mathsf{We}(\alpha,\beta)$	$f(x)=\alpha\beta\,x^{\alpha-1}\,e^{-\beta\,x^\alpha}$	$x\in\mathbb{R}_+$	$\frac{\Gamma(1+\alpha^{-1})}{\beta^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)}{\beta^{2/\alpha}}$	