# Sta 532: Homework #10

**Problem 1:** Let $\{X_i : 1 \le i \le n\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu, \tau_0^{-1})$ be normally distributed with known precision $\tau_0$ and uncertain mean $\mu$, and let $\mu \sim \mathsf{No}(\xi_0, \lambda_0^{-1})$ have a normal prior distribution with known mean $\xi_0$ and precision $\lambda_0$. Show that, given $\{X_1, \ldots, X_n\}$, the conditional distribution of $X_{n+1}$ is normal with mean

$$\frac{n\tau_0 \bar{X}_n + \lambda_0 \xi_0}{n\tau_0 + \lambda_0}$$

and variance

$$\frac{1}{\tau_0} + \frac{1}{n\tau_0 + \lambda_0}$$

**Problem 2:** Let $\mathbf{x} = \{X_i\} \overset{\text{ind}}{\sim} \mathsf{Po}(\theta_i)$ be Poisson variables and let $\{\theta_i\} \overset{\text{iid}}{\sim} \mathsf{Ga}(\alpha, \beta)$ have Gamma prior distributions.

1. Show that the posterior (*i.e.*, conditional on $\mathbf{x}$) distribution for $\theta_i$ is also $\mathsf{Ga}(\alpha_i^\star, \beta_i^\star)$, and find the updated parameters $\alpha_i^\star$ and $\beta_i^\star$;
2. Find the posterior mean $\mathsf{E}[\theta_i \mid \mathbf{x}]$, for fixed $\alpha$ and $\beta$;
3. Show that $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{NB}(\alpha, p)$ have a negative binomial marginal distribution, and find the value of $p$.
4. Find estimates $\hat{\alpha}, \hat{\beta}$, either MLEs or method-of-moments estimators based on the sample mean and variance $\bar{X}_n$ and $S^2 = \frac{1}{n}\sum(X_i - \bar{X}_n)^2$.
5. Give the EB estimates $\delta_i(\mathbf{x})$ of $\theta_i$.

Last edited: April 11, 2015