Sta 961: Homework #2

1. Equation (1) of the class notes *Introduction to Computer Experiments* gives explicit expressions for the Matérn correlation function $r_\nu(h \mid \theta)$ for odd half-integer $\nu = n+1/2$ smoothness parameter, for $n = 1$ and $n = 2$, as the products of exponential function and a polynomial in h of degree n. Do the same for $n = 3$ and $n = 4$, i.e., $\nu = 7/2$ and $\nu = 9/2$.

2. Let $\mathbf{x} = (-1, -0.96, \cdots, 0.92, 0.96, 1)$ be a vector of $n = 51$ equally spaced points on the interval $\mathcal{X} := [-1, 1]$. Draw and plot 10 independent replicates of a Gaussian stochastic process $Y(x)$ on $\mathcal{X}$ with mean zero and Matérn covariance function (see "covar.pdf" class notes) with parameters $\theta_1 = 1$, $\theta_2 = 0.25$, and $\theta_3 = 3/2$. Also, overlay with dashed lines at $\pm 1.96\sigma$, a pointwise 95% band.

3. As above, but now draw and plot 10 replicates of the *conditional* distribution of the same process $Y(x)$, *given* the values $Y(x) = 2x + \cos(2\pi x)$ at $x \in \{(i-n)/n,\ 1 \le i \le 2n-1\}$ for $n = 3$ and $n = 5$. Overlay with dashed lines at $\pm 1.96\sigma(x)$, a pointwise 95% envelope, where now $\sigma^2(x)$ will depend on x.

4. If the time series X_t satisfies

$$X_t - (5/6)X_{t-1} + (1/6)X_{t-2} = \zeta_t$$

for $\{\zeta_t\} \overset{\text{iid}}{\sim} \mathsf{No}(0, 1)$, and if

$$X_5 = f \quad X_6 = e \quad X_7 = d \quad X_8 = c \quad X_9 = b \quad X_{10} = a$$

for specified real numbers a, b, c, d, e, f, find the conditional distribution (including the values of any parameters) of X_{12} given $\mathcal{F}_{10}$, *i.e.*, given $\{X_s : \ s \le 10\}$.

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