Unit 2: Probability and distributions

1. Probability and conditional probability

Sta 101 – Spring 2019

Duke University, Department of Statistical Science

Dr. Ellison

Slides posted at https://www2.stat.duke.edu/courses/Spring19/sta101.001/

Outline

1. Housekeeping
2. Readiness assessment
3. Main ideas
   1. Differences between Probability Properties
      1. Disjoint and independent do not mean the same thing
   2. Relationships between Probability Properties
      1. Application of the addition rule depends on disjointness of events
      2. Bayes' theorem works for all types of events
4. Summary

Announcements

Coming up...

- Lab Assignment 2 is due **Thursday just before your lab section time.**
- Start working on Problem Set 2.

1

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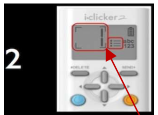
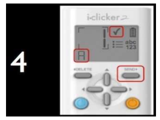
Clicker Help in Readiness Assessment



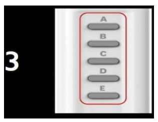
Syncing to the Quiz

You should see this screen once I press the RA clicker start button (I'll tell you when). If not, press the blue refresh button!



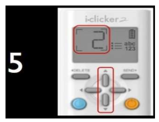
Reviewing/Changing your Answer

The most recent letter you chose should show up here. If you want to change your answer, just press the letter you want. My iclicker box stores your most recent letter you pressed for that number.



Submitting your Answer

Select the **letter** you want to pick for the number shown here.



Moving to Other Questions

Press the **up button** to go to higher numbers. Press the **down button** to go lower numbers.

1

Important Reminder

for Scratch Cards and Application Exercises

Accurately taking group attendance on the scratch cards and application exercises is part of your participation grade!

Correct Name Format

Group Name: "Stats IS FUN"

Present Group Members:
- Amy Lastname
- Barry Lastname

Absent Group Members:
- Missing Mary
- Absent Albert

Incorrect Name Format

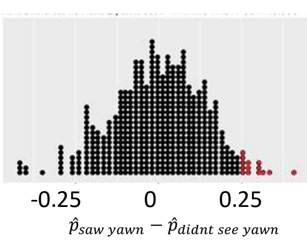
"Stats IS FUN"

- Amy Lastname
- Barry Lastname
- Missing Mary
- Absent Albert

Outline

Making an Inference

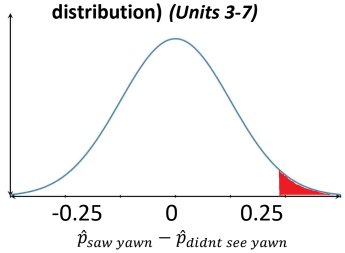
Randomization Distribution *(Unit 1)*



Vs.

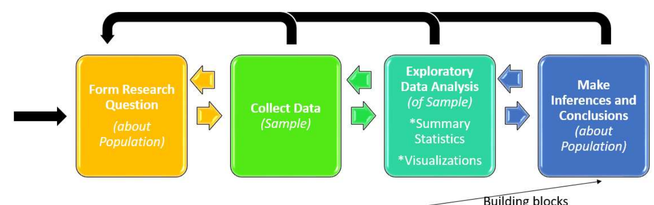
Sampling Distribution

(Special kind of normal distribution) ***(Units 3-7)***



Outline

Course Overview



- Mathematics behind statistics
  - *Unit 2 - Probability & distributions:*
    - Basics of probability and chance processes
    - Bayesian perspective in statistical inference
    - The normal and binomial distributions.

Outline

Do mutually exclusive/disjoint events, independent events, or dependent events mean same thing?

Outline

1. Housekeeping

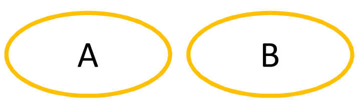
2. Readiness assessment

3. Main ideas
   1. **Differences** between Probability Properties
      1. Disjoint and independent do not mean the same thing
   2. **Relationships** between Probability Properties
      1. Application of the addition rule depends on disjointness of events
      2. Bayes' theorem works for all types of events

4. Summary

1. Disjoint and independent do not mean the same thing

- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But they might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A \text{ and } B) = 0$



3

1. Disjoint and independent do not mean the same thing

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  - For disjoint A and B: $P(A \text{ and } B) = 0$

- If A and B are *independent events*, having information on A does not tell us anything about B (and vice versa)

A and B are **independent** ⟺ ***ALL of these equations hold:***
- $P(A \mid B) = P(A)$
- $P(B \mid A) = P(B)$
- $P(A \text{ and } B) = P(A) \times P(B)$

3

1. Disjoint and independent do not mean the same thing

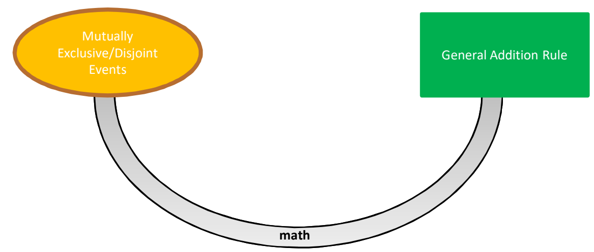
- *Disjoint (mutually exclusive) events* cannot happen at the same time
  - A voter cannot register as a Democrat and a Republican at the same time
  - But they might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A \text{ and } B) = 0$
- If A and B are *dependent events*, having information on A DOES tell us anything about B (and vice versa)

A and B are dependent ⟺ ***NONE of these equations hold:***

- $P(A \mid B) = P(A)$
- $P(B \mid A) = P(B)$
- $P(A \text{ and } B) = P(A) \times P(B)$

3

How are Mutually Exclusive/Disjoint Events and the General Addition Rule related?



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   2. **Relationships between Probability Properties**
      1. Application of the addition rule depends on disjointness of events
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4. Summary

2. Application of the addition rule depends on disjointness of events

- *General addition rule:* P(A or B) = P(A) + P(B) - P(A and B)
- A or B = either A or B or both

4

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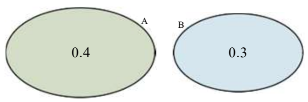
disjoint events:
P(A or B)
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0 = 0.7



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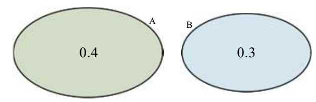

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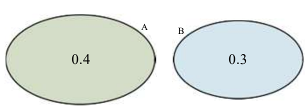
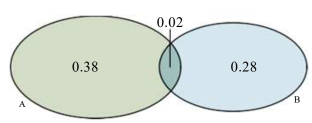


non-disjoint events:
P(A or B)
= P(A) + P(B) - P(A and B)
= 0.4 + 0.3 - 0.02 = 0.68



4

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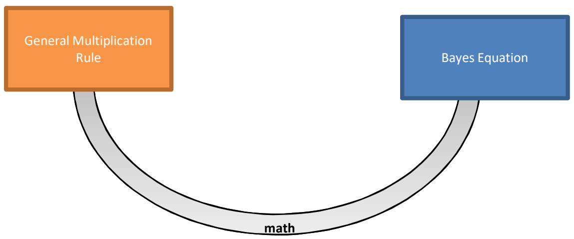
How are Bayes Equation and the General Multiplication Rule related?



3. Bayes' theorem works for all types of events

- *Bayes' theorem/equation:* $P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$

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disjoint events:

- We know P(A | B) = ?,
  since if B happened A
  could not have happened

*assume P(B)≠0

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independent events:

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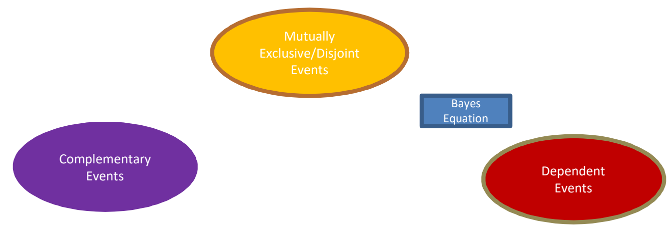
  = *P(A)* × P(B)

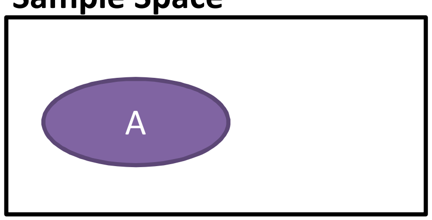
5

How do complementary, mutually exclusive/disjoint, and dependent events relate?



How do complementary and mutually exclusive/disjoint events relate?

Sample Space



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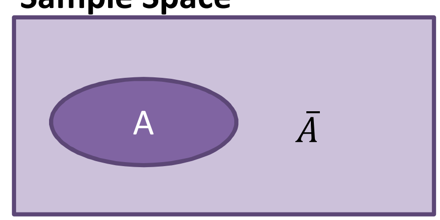
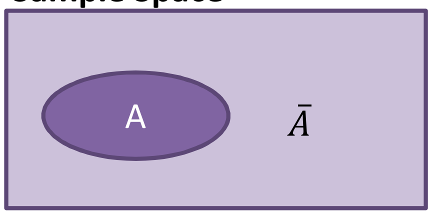


A and $\bar{A}$ are **complementary events**.

$P(A \text{ and } \bar{A}) = ?$

How do complementary and mutually exclusive/disjoint events relate?

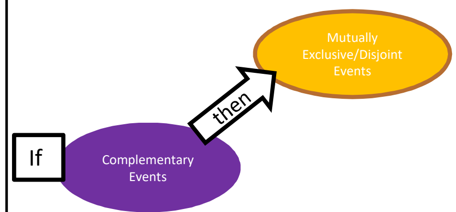
Sample Space



A and $\bar{A}$ are **complementary events**.

$P(A \text{ and } \bar{A})=0$

How do complementary and mutually exclusive/disjoint events relate?



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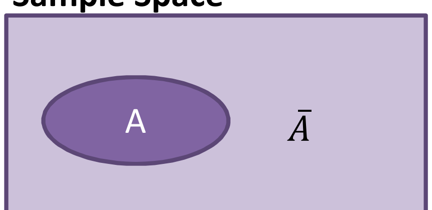


A and $\bar{A}$ are **complementary events**.

$P(A \text{ and } \bar{A})=0$

Useful equation

$P(A) + P(\bar{A})=1$

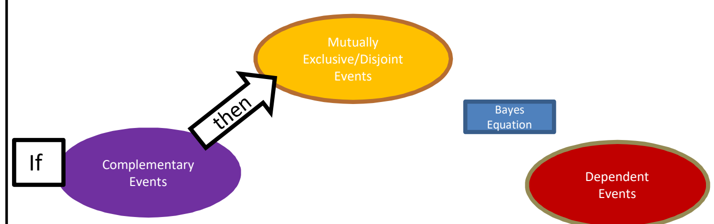
Where does it come from?

$P(A \text{ or } \bar{A})=1$

0

$P(A \text{ or } \bar{A})=P(A) + P(\bar{A}) - \sout{P(A \text{ and } \bar{A})}$

How do complementary, mutually exclusive/disjoint, and dependent events relate?



How do mutually exclusive/disjoint and dependent events relate?

Mutually Exclusive/Disjoint Events

Bayes Equation

Dependent Events

And both events have probabilities ≠ 0

$$P(A|B) = \frac{P(A\ and\ B)}{P(B)}$$

How do mutually exclusive/disjoint and dependent events relate?

Mutually Exclusive/Disjoint Events

Bayes Equation

Dependent Events

And both events have probabilities ≠ 0

$$P(A|B) = \frac{P(A\ and\ B)}{P(B)} = \frac{0}{P(B)} = 0$$

How do mutually exclusive/disjoint and dependent events relate?

Mutually Exclusive/Disjoint Events

Bayes Equation

Dependent Events

And both events have probabilities ≠ 0

$$P(A|B) = 0$$

$$P(A) \neq 0$$

How do mutually exclusive/disjoint and dependent events relate?

Mutually Exclusive/Disjoint Events

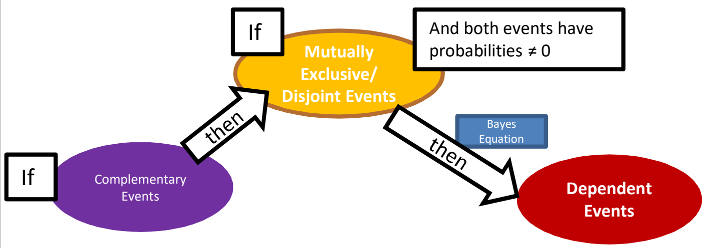
Bayes Equation

Dependent Events

And both events have probabilities ≠ 0

$$P(A) \neq P(A|B)$$

How do complementary, mutually exclusive/disjoint, and dependent events relate?



Application exercise: 2.1 Probability and conditional probability

See the course website for instructions.

Summary of main ideas

1. Disjoint and independent do not mean the same thing
2. Application of the addition rule depends on disjointness of events
3. Bayes' theorem works for all types of events