STA 113 Name:________________________

Probability and Statistics First Test

2:15-3:30 pm Thursday, February 8, 1996

This is a **closed-book** examination, so please do not refer to your notes, the text, or to any other books. A page of formulas, and two blank work sheets, are appended to the exam. Please do not use your own scratch paper; you may use your own calculator but may not share one.

If you don't understand something in one of the questions feel free to ask me, but please do not talk to each other. Please sign the Duke Honor Code:

I have neither given nor received aid on this examination.

____

signature

It is to your advantage to write your solutions as clearly as possible, and to **show** your **work**—*unsupported answers are not acceptable.* Good luck.

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1. Each year, students of Probability State University enter a lottery in which they are given a $1/3$ chance of winning the right to buy coveted seasons-tickets to watch the Prob/Stat Bowling Team play. Rocko, a Business Administration student, calculates that in his four years of study he has altogether a $4 \times 1/3 = 4/3$ chance of winning tickets, a "statistical guarantee."

   a. Four-thirds seems a bit high to me. What do *you* think Rocko's chances are of winning tickets in at least one of his four years?

      Pr[Tickets] = ____________

   b. What assumptions did you use in computing your answer above?

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2. One of the first published uses of the Poisson distribution was an 1898 study of accidental deaths due to kicks from horses, from twenty years' data on Prussian cavalry units. The data are *very* well fit by a Poisson distribution with a mean of $\lambda = 0.7$ annual deaths per army unit. For all three parts of this question you may assume that this Poisson distribution is exactly correct.

a. What is the probability the First Army Unit would experience zero deaths in a particular year, if the number X_1 of deaths follows this distribution? Give both an exact mathematical expression and also a numerical answer accurate to three decimal places:

$\Pr[X_1 = 0] =$ ____________ $\approx$ 0.____________

b. The data set includes annual death counts attributed to horse-kicks for 280 Prussian units. About how many of those should have experienced zero annual deaths? Give a numerical answer:

$N_0 \approx$ ____________

c. About how many deaths in all would you expect in this data, covering the 280 units? Give a numerical answer:

$\mathsf{E}[X_1 + \ldots + X_{280}] \approx$ ____________

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3. A bag holds a dozen dynamic random-access memory (DRAM) chips, each with a color-coded band indicating the chip's maximum speed (measured in nanoseconds-per-cycle): six white (for 80ns memory), four red (70ns), and two blue (60ns). Three chips are drawn at random from the bag, without replacement. All three chips could be the same color (*i.e.*, same speed); they could all be different colors; or two chips might be one color and the other chip another. Let X be the total number of different colors of chips among the three chips drawn, with possible values $\{1, 2, 3\}$.

Which is more likely: that all three are the same color, or that they are all different colors? Show your work (which should include computation of both probabilities).

Circle one: All Same All Different Reasoning:

Check: The third possibility, that the three chips include two different colors, has probability $\Pr[X = 2] = 37/55$.

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4. Radioactive leakage may occur at a nuclear power plant. If there is a leakage, a first alarm sounds with probability 0.95. Moreover, given that there is a leakage and that the first alarm fails, a backup alarm will sound with probability 0.90. Also, the first alarm may sometime sound when there is no leakage: the probability of this event is 0.1. The probability of a leakage is 0.04.

a. What is the probability that there is no leakage?

$\Pr[\textsf{No Leakage}] =$ ____________

b. What is the probability that, given that there is a leakage, neither alarm will sound?

$\Pr[\textsf{No Alarm}|\textsf{Leakage}] =$ ____________

c. Define *safe operation* as the event in which there is no leakage or there is a leakage and it is detected by one of the two alarms. What is the probability of "safe operation"?

$\Pr[\textsf{Safe Operation}] =$ ____________

d. What is the probability that there actually is a leakage, given that the first alarm sounds?

$\Pr[\textsf{Leakage}|1^{st}\textsf{Alarm}] =$ ____________

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5. The random variable X has a density function equal to kx^2 between minus one and and plus two ($i.e.$, on $(-1,2)$), for some constant $k>0$, and zero everywhere else.

a. What is the value of k?

$k =$ ____________

b. What is the expected value of the square of X?

$\mathsf{E}[X^2] =$ ____________

c. What are the mean and variance of X?

$\mathsf{E}[X] =$ ____________ $\mathsf{V}[X] =$ ____________

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Formulas, By Request

$$(n)_k = \frac{n!}{(n-k)!} = \overbrace{(n)(n-1)\cdots(n-k+1)}^{k \text{ terms}}$$

$$\text{"}n \text{ choose } k\text{":} \quad \binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{(n)(n-1)\cdots(n-k+1)}{(k)(k-1)\cdots(1)}$$

Distributions

Binomial	$\Pr[X=x] = \binom{n}{x} p^x q^{(n-x)},\ x = 0, 1, \ldots, n$	$\mu = np,\ \sigma^2 = npq$
Poisson	$\Pr[X=k] = \lambda^k\, e^{-\lambda}/k!,\ k = 0, 1, \ldots$	$\mu = \lambda,\ \sigma^2 = \lambda$
Geometric	$\Pr[Y=y] = pq^{y-1},\ y = 1, 2, \ldots$	$\mu = 1/p,\ \sigma^2 = q/p^2$
Neg. Binomial	$\Pr[Y=y] = \binom{y-1}{r-1} p^r q^{y-r},\ y = r, r{+}1, \ldots$	$\mu = r/p,\ \sigma^2 = rq/p^2$
Hypergeometric	$\Pr[Y=y] = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}},\ y = 0, 1, \ldots, n$	$\mu = \frac{nr}{N},\ \sigma^2 = \text{a mess}$
Exponential	$\Pr[X \le x] = 1 - e^{-x/\lambda},\ x \in (0, \infty)$ $f(x) = \frac{1}{\lambda} e^{-x/\lambda},\ x > 0;\ 0,\ x \le 0$	$\mu = \lambda,\ \sigma^2 = \lambda^2$
Uniform	$\Pr[X \le x] = (x-a)/(b-a),\ x \in (a, b)$ $f(x) = \frac{1}{b-a},\ a{<}x{<}b;\ 0,\ x \notin (a, b)$	$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$

$$\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = (a+b)^n \qquad \int_0^\infty t^n e^{-t}\, dt = n!$$

$$\sum_{k=0}^{\infty} ar^k = a/(1-r) \qquad \sum_{k=0}^{n} ar^k = (a - ar^{n+1})/(1-r)$$

$$\Pr[X \le x] = F(x) = \int_{-\infty}^{x} f(t)\, dt \qquad f(x) = F'(x)$$

$$\mathsf{E}[g(X)] = \int g(x) f(x)\, dx \qquad \mathsf{E}[g(X)] = \sum_x g(x) P(x)$$

$$\mu = \mathsf{E}[X] = \int_{-\infty}^{\infty} x f(x)\, dx \qquad \sigma^2 = \mathsf{E}[(X-\mu)^2] = \mathsf{E}[X^2] - \mu^2$$

A **and** $B : A \cap B$ **not** $A : A^c$ A **or** $B : A \cup B$

$$\mathsf{P}[A|B] = \mathsf{P}[A \cap B]/\mathsf{P}[B] \qquad \mathsf{P}[B|A] = \mathsf{P}[A \cap B]/\mathsf{P}[A]$$

$$\mathsf{P}[A \cap B] = \mathsf{P}[A|B]\, \mathsf{P}[B] \qquad \mathsf{P}[A \cup B] = \mathsf{P}[A] + \mathsf{P}[B] - \mathsf{P}[A \cap B]$$

Independent $\Leftrightarrow \mathsf{P}[A \cap B] = \mathsf{P}[A]\, \mathsf{P}[B]$; Exclusive $\Leftrightarrow \mathsf{P}[A \cap B] = 0$

Bayes: $\mathsf{P}[A_i|B] = \frac{\mathsf{P}[B|A_i]\mathsf{P}[A_i]}{\mathsf{P}[B|A_1]\mathsf{P}[A_1] + \ldots + \mathsf{P}[B|A_k]\mathsf{P}[A_k]}$ $A_i \cap A_j = \emptyset,\quad \mathsf{P}[A_1] + \ldots + \mathsf{P}[A_k] = 1$

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(Nearly) Blank Work-Sheet #1 (of 2)

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(Nearly) Blank Work-Sheet #2 (of 2)