STA 113 Name:________________________

Probability and Statistics in Engineering: Final Examination

7:00-10:00 pm Tuesday, May 6, 1997

This is a **closed-book** examination, so please do not refer to your notes, the text, or to any other books. A few pages of formulas, tables, and blank work-sheets are appended to the exam. Please do not use your own scratch paper; you may use your own calculator but may not share one. Each problem counts the same (25 points), but they may not seem equally long or equally difficult. Please **DO NOT** spend time redoing calculations already done for you (e.g. sample means & variances). As usual, **pictures** (sketches, plots, etc.) are helpful for most problems.

Please ask me to explain anything you don't understand in the questions (wording, definitions, *etc.*). It is to your advantage to write your solutions as **clear**ly as possible, and to **show** enough of your **work** to make clear how you got your answer (especially important if you used a calculator in some key step)— even correct answers may be awarded little or **no credit** without a clear presentation of your method. Good luck! Please sign the Duke Honor Code:

I have neither given nor received aid on this examination.

____

signature

1.	/25
2.	/25
3.	/25
4.	/25
5.	/25
6.	/25
	/150

STA 113

Probability and Statistics in Engineering: Final Examination

1. The data below are from a study of the causes of vacuum tube (VT) failure in World War II artillery shells. Each failure was classified twice: once for the Position of the failure (A_1 for Top, A_2 for Bottom) and once for the Type of failure (three types were distinguished, which we'll call B_1, B_2, B_3):

Position	*Type of failure*			
in shell	$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	**Total**
A_1 = Top	75	10	15	100
A_2 = Bottom	40	30	10	80
Total	115	40	25	180

a. (5pt) Which statistical method would be best for testing the hypothesis that failure type and position are independent?

- ∘ Paired t test
- ∘ Confidence Interval
- ∘ Simple linear regression
- ∘ Unpaired t test
- ∘ ANOVA F test
- ∘ Multiple linear regression
- ∘ χ^2 test of homogeneity and independence
- ∘ Nonlinear regression

Here is a Minitab output:

```
Expected counts are printed below observed counts
              B1       B2       B3     Total
    1         75       10       15       100
           63.89    22.22    13.89

    2         40       30       10        80
           51.11    17.78    11.11

Total        115       40       25       180

ChiSq   = 1.932  + 6.722  + 0.089
        + 2.415  + 8.403  + 0.111  = 19.673

   df      = 2
```

b. (10+5pt) Find the approximate P-value for a test of this hypothesis, and indicate whether or not you would reject the hypothesis at level $\alpha = .01$:

$P \approx$ ______________ Circle one: Accept Reject

c. (5pt) What do you conclude about VT failure in WW-II artillery shells?

Probability and Statistics in Engineering: Final Examination

2. In a weight loss experiment 10 men followed Diet A and 7 men followed Diet B. The weight losses after one month are displayed in the following table; you may assume that the data are normally distributed with a common variance:

Weight loss (lb)		*Weight loss (lb)*	
Diet A	**Diet B**	**Diet A**	**Diet B**
15.3	3.4	10.0	5.2
2.1	10.9	8.3	2.5
8.8	2.8	9.4	
5.1	7.8	12.5	
8.3	0.9	11.1	

a. (5pt) Which statistical method would be best for testing to see if the two diets differ in their effectiveness?

- ∘ Paired t test
- ∘ Confidence Interval
- ∘ Simple linear regression
- ∘ Unpaired t test
- ∘ ANOVA F test
- ∘ Multiple linear regression
- ∘ χ^2 test of homogeneity and independence
- ∘ Nonlinear regression

b. (20pt) Describe in detail the null and alternative hypotheses and determine (using α=.05) which diet is more effective in losing weight. You may find some of the following useful:

$\Sigma X_A = 90.9$; $\Sigma {X_A}^2 = 947.55$; $\Sigma(X_A - 90.9/10)^2 = 121.269$;
$\Sigma X_B = 33.5$; $\Sigma {X_B}^2 = 233.15$; $\Sigma(X_B - 33.5/7)^2 = 72.829$;
$\Sigma X = 124.4$; $\Sigma X^2 = 1180.7$; $\Sigma(X - 124.4/17)^2 = 270.38$.

Probability and Statistics in Engineering: Final Examination

3. The data below record the shear strengths (y) of spot welds of 0.040" gauge steel, corresponding to weld diameters (x) measured in units of thousandths of an inch:

y :	350	380	385	450	465	485	535	555	590	605
x :	140	155	160	165	175	165	195	185	195	210

Here is the `Minitab` output for a regression run:

```
      MTB > read 'shear.dat' into c1-c2
Entering data from file: shear.dat
10 rows read.
MTB > name c1='y' c2='x'
MTB > regress 'y' 1 'x'
The regression equation is
y = - 216 + 3.99 x

Predictor      Coef    Stdev  t-ratio       p
Constant    -216.33    80.51    -2.69   0.028
x            3.9904   0.4583     8.71   0.000
      s = 29.60    R-sq = 90.5%     R-sq(adj) = 89.3%
Analysis of Variance

SOURCE       DF      SS      MS       F       p
Regression    1   66440   66440   75.83   0.000
Error         8    7010     876
Total         9   73450
```

a. (5pt) Which statistical method would be best for predicting shear strength as a function of weld diameter?
   - ○ Paired t test ○ Confidence Interval ○ Simple linear regression
   - ○ Unpaired t test ○ ANOVA F test ○ Multiple linear regression
   - ○ χ^2 test of homogeneity and independence ○ Nonlinear regression

b. (5pt) Find the predicted strength of a weld with diameter 0.160":

   $E[Y|X = 160] \approx$ ______________

c. (15pt) Give a 90% confidence interval for the slope β_1, the change in weld strength for each one-mil increase in weld diameter:

   $\beta_1 =$ ______________ $\pm$ ______________

4. The number of trials needed for the first success in a sequence of independent trials, all with the same success probability p (and also the same failure probability $q = 1 - p$), will follow the geometric distribution given by $\mathsf{P}[X_j = x] = p\,q^{x-1}$ for $x = 1, 2, \dots$ If we observe n observations $X_1, \dots, X_n$,

a. (10pt) Find the maximum likelihood estimator (MLE) for p:

   $\hat{p} =$ ______________

b. (5pt) What are the mean and variance of ONE OVER this estimate, as a function of n and the true but unknown success probability p? Why? (The Formula Sheet may be helpful)

   $\mathsf{E}[1/\hat{p}] =$ ______________ $\mathsf{V}[1/\hat{p}] =$ ______________

c. (5pt) If we observe $n = 100$ failure counts $X_1, \cdots, X_{100}$, with sum $\Sigma X_j = 900$, give an approximate 90% confidence interval for $1/p$:

   $1/p =$ ______________ $\pm$ ______________

d. (5pt) Use the interval you found in c. above to give an approximate 90% confidence interval for p itself:

   $p \in ($______________, ______________$)$

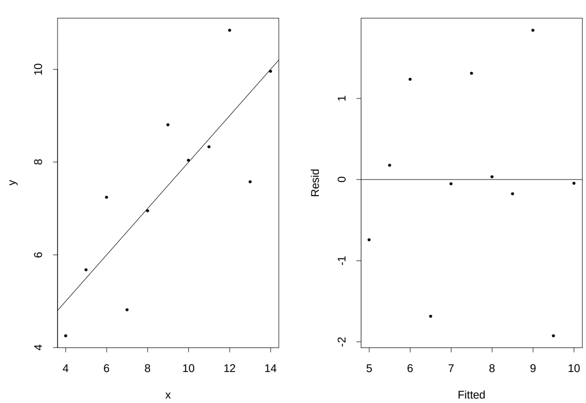
5. Here are scattergrams and residual plots from the four Anscombe data sets; each has the same Minitab output, Given below. For each, tell what Linear Regression assumption appears to be violated (if any) and BRIEFLY suggest a possible remedy:

```
      MTB > read 'anscombe' into c1-c2
Entering data from file: anscombe
11 rows read.
MTB > name c1 'X' c2 'Y'
MTB > regress c2 1 c1;
SUBC> fits = c3;
SUBC> resid = c4.

      The regression equation is
Y = 3.00 + 0.500 X

Predictor     Coef    Stdev  t-ratio       p
Constant     3.000    1.125     2.67   0.026
X           0.5000   0.1179     4.24   0.002
      s = 1.237    R-sq = 66.7%     R-sq(adj) = 62.9%
Analysis of Variance

SOURCE      DF      SS      MS      F      p
Regression   1  27.510  27.510  17.99  0.002
Error        9  13.763   1.529
Total       10  41.273
```

a. (5pt) What (if anything) from the `MINITAB` output above suggested that there may be some problem using linear regression with this data?

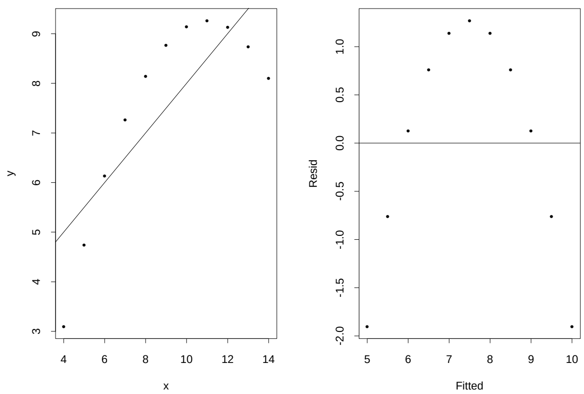
Check one: ○ Nothing ○ Something: (what?)

b. (5pt) Which (if any) assumption(s) are violated? What remedy would you suggest?

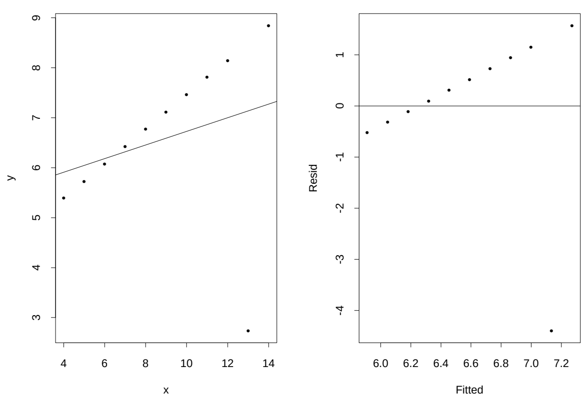


5. (Continued)

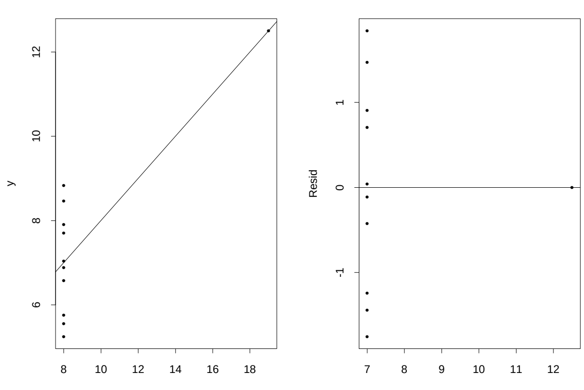
c. (5pt) Which (if any) assumption(s) are violated? What remedy would you suggest?



d. (5pt) Which (if any) assumption(s) are violated? What remedy would you suggest?



e. (5pt) Which (if any) assumption(s) are violated? What remedy would you suggest?



Probability and Statistics in Engineering: Final Examination

6. In the week following final exams, 10% of Duke students go to the beach (it always seems like more); 50% go home; and 40% start a job. Of the beach-goers, 90% (claim to) have fun; 22% of the home-goers have fun; and a disappointing 25% of the workers have fun.

a. (5pt) What is the probability that a randomly-selected Duke student will go to the beach and not have fun?

$P =$ ____________

b. (5pt) What is the probability that a randomly-selected Duke student will have fun (anywhere)?

$P =$ ____________

c. (10pt) What is the probability that a randomly-selected Duke student who IS having fun can see the ocean?

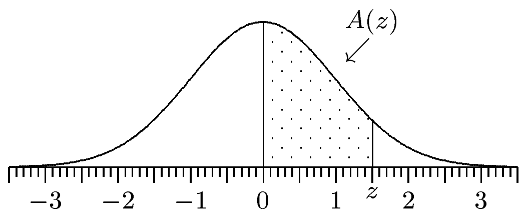
$P =$ ____________

d. (5pt) Don't you wish you had a job working at home and that you lived at the beach?

Check one: ○ Yes ○ I didn't understand the question

TABLE 4 Normal Curve Areas

$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE 7 Critical Values

$A(1.282) = 0.4000$ $A(1.645) = 0.4500$ $A(1.960) = 0.4750$ $A(2.326) = 0.4900$

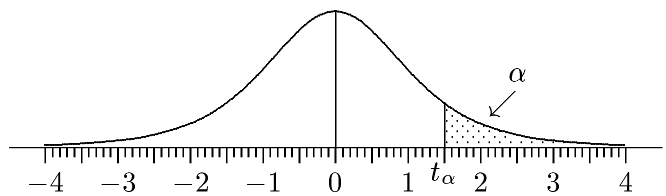
$A(2.576) = 0.4950$ $A(3.090) = 0.4990$ $A(3.291) = 0.4995$ $A(3.719) = 0.4999$

STA 113

Probability and Statistics in Engineering: Final Examination

TABLE 7 Critical Values for Student's t

$$\alpha = \int_{t_\alpha}^{\infty} \frac{c\ dt}{(1 + t^2/\nu)^{(\nu+1)/2}}$$



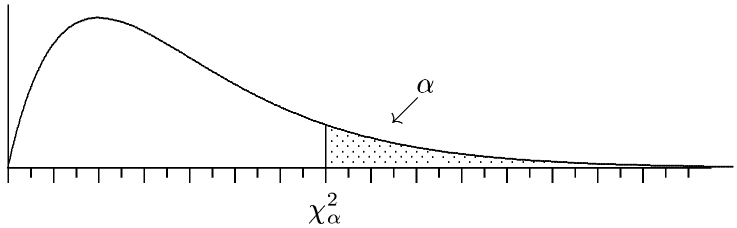
ν	$t_{.40}$	$t_{.30}$	$t_{.20}$	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$	$t_{.0001}$
1	0.325	0.727	1.376	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

STA 113

Probability and Statistics in Engineering: Final Examination

TABLE 8 **Critical Values of χ^2**

$$\alpha = \int_{\chi^2_\alpha}^{\infty} c\, x^{\nu/2-1} e^{-x/2}\, dx$$



ν	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	$\chi^2_{.001}$	$\chi^2_{.0005}$	$\chi^2_{.0001}$
1	0.4549	1.3233	2.7055	3.8415	5.0239	6.6349	7.87940	10.8276	12.1157	15.1367
2	1.3863	2.7726	4.6052	5.9915	7.3778	9.2103	10.5966	13.8155	15.2018	18.4207
3	2.3660	4.1083	6.2514	7.8147	9.3484	11.3449	12.8382	16.2662	17.7300	21.1075
4	3.3567	5.3853	7.7794	9.4877	11.1433	13.2767	14.8603	18.4668	19.9974	23.5127
5	4.3515	6.6257	9.2364	11.0705	12.8325	15.0863	16.7496	20.515	22.10530	25.7448
6	5.3481	7.8408	10.6446	12.5916	14.4494	16.8119	18.5476	22.4577	24.1028	27.8563
7	6.3458	9.0371	12.0170	14.0671	16.0128	18.4753	20.2777	24.3219	26.0178	29.8775
8	7.3441	10.219	13.3616	15.5073	17.5345	20.0902	21.9550	26.1245	27.8680	31.8276
9	8.3428	11.389	14.6837	16.9190	19.0228	21.6660	23.5894	27.8772	29.6658	33.7199
10	9.3418	12.549	15.9872	18.3070	20.4831	23.2092	25.1882	29.5883	31.4198	35.5640
11	10.341	13.701	17.2750	19.6751	21.9200	24.7249	26.7568	31.2641	33.1366	37.3670
12	11.340	14.845	18.5493	21.0260	23.3366	26.2169	28.2995	32.9095	34.8213	39.1344
13	12.340	15.984	19.8119	22.3620	24.7356	27.6882	29.8195	34.5282	36.4778	40.8707
14	13.339	17.117	21.0641	23.6848	26.1189	29.1412	31.3193	36.1233	38.1094	42.5793
15	14.339	18.245	22.3071	24.9958	27.4884	30.5779	32.8013	37.6973	39.7188	44.2632
16	15.338	19.369	23.5418	26.2962	28.8453	31.9999	34.2672	39.2524	41.3081	45.9249
17	16.338	20.489	24.7690	27.5871	30.1910	33.4087	35.7185	40.7902	42.8792	47.5664
18	17.338	21.605	25.9894	28.8693	31.5264	34.8053	37.1565	42.3124	44.4338	49.1894
19	18.338	22.718	27.2036	30.1435	32.8523	36.1909	38.5823	43.8202	45.9731	50.7955
20	19.337	23.828	28.4120	31.4104	34.1696	37.5662	39.9968	45.3147	47.4985	52.3860
21	20.337	24.935	29.6151	32.6706	35.4789	38.9322	41.4011	46.7970	49.0108	53.9620
22	21.337	26.039	30.8133	33.9244	36.7807	40.2894	42.7957	48.2679	50.5111	55.5246
23	22.337	27.141	32.0069	35.1725	38.0756	41.6384	44.1813	49.7282	52.0002	57.0746
24	23.337	28.241	33.1962	36.4150	39.3641	42.9798	45.5585	51.1786	53.4788	58.6130
25	24.337	29.339	34.3816	37.6525	40.6465	44.3141	46.9279	52.6197	54.9475	60.1403
26	25.336	30.435	35.5632	38.8851	41.9232	45.6417	48.2899	54.0520	56.4069	61.6573
27	26.336	31.528	36.7412	40.1133	43.1945	46.9629	49.6449	55.4760	57.8576	63.1645
28	27.336	32.620	37.9159	41.3371	44.4608	48.2782	50.9934	56.8923	59.3000	64.6624
29	28.336	33.711	39.0875	42.5570	45.7223	49.5879	52.3356	58.3012	60.7346	66.1517
30	29.336	34.800	40.2560	43.7730	46.9792	50.8922	53.6720	59.7031	62.1619	67.6326
40	39.336	45.616	51.8051	55.7585	59.3417	63.6907	66.7660	73.4020	76.0946	82.0623
50	49.335	56.334	63.1671	67.5048	71.4202	76.1539	79.4900	86.6608	89.5605	95.9687
60	59.335	66.981	74.3970	79.0819	83.2977	88.3794	91.9517	99.6072	102.695	109.503
70	69.335	77.577	85.5270	90.5312	95.0232	100.425	104.215	112.317	115.578	122.755
80	79.334	88.130	96.5782	101.879	106.629	112.329	116.321	124.839	128.261	135.782
90	89.334	98.650	107.565	113.145	118.136	124.116	128.299	137.208	140.782	148.627
100	99.334	109.14	118.498	124.342	129.561	135.807	140.169	149.449	153.167	161.319

STA 113 Name: ______________________________

Probability and Statistics in Engineering: Final Examination

(Nearly) Blank Work-Sheet #1 (of 2)

Probability and Statistics in Engineering: Final Examination

(Nearly) Blank Work-Sheet #2 (of 2)

STA 113 Name:________________________

Probability and Statistics in Engineering: Final Examination

Distributions

Bernoulli	$\Pr[X=x] = p^x q^{1-x},\ x=0,1$	$\mu = p,$	$\sigma^2 = pq$
Binomial	$\Pr[X=x] = \binom{n}{x} p^x q^{n-x},\ x=0,1,\ldots,n$	$\mu = np,$	$\sigma^2 = npq$
Poisson	$\Pr[X=x] = \lambda^x e^{-\lambda}/x!,\ x=0,1,\ldots$	$\mu = \lambda,$	$\sigma^2 = \lambda$
Geometric	$\Pr[X=x] = pq^{x-1},\ x=1,2,\ldots$	$\mu = 1/p,$	$\sigma^2 = q/p^2$
Neg. Binomial	$\Pr[X=x] = \binom{x-1}{r-1} p^r q^{x-r},\ x=r,r{+}1,\ldots$	$\mu = r/p,$	$\sigma^2 = rq/p^2$
Hypergeometric	$\Pr[X=x] = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}},\ x=0,1,\ldots,n$	$\mu = \frac{nr}{N},$	$\sigma^2 =$ a mess
Exponential	$\Pr[X \le x] = 1 - e^{-x/\beta},\ x \in (0,\infty)$ $f(x) = \frac{1}{\beta} e^{-x/\beta},\ x>0;\ 0,\ x \le 0$	$\mu = \beta,$	$\sigma^2 = \beta^2$
Gamma	$\Pr[X \le x] = 1 - e^{-x/\beta} \sum_{k=0}^{\alpha-1} (x/\beta)^k/k!,\ x>0$ $f(x) = \frac{x^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} e^{-x/\beta},\ x>0;\ 0,\ x \le 0$	$\mu = \alpha\beta,$	$\sigma^2 = \alpha\beta^2$
Uniform	$\Pr[X \le x] = (x-a)/(b-a),\ x \in (a,b)$ $f(x) = \frac{1}{b-a},\ a<x<b;\ 0,\ x \notin (a,b)$	$\mu = \frac{a+b}{2},$	$\sigma^2 = \frac{(b-a)^2}{12}$
Normal	$\Pr[X \le x] = \Phi(\frac{x-\mu}{\sigma}) = \begin{cases} .5 + A(\frac{\lvert x-\mu\rvert}{\sigma}) & \text{if } x>\mu \\ .5 - A(\frac{\lvert x-\mu\rvert}{\sigma}) & \text{if } x<\mu \end{cases}$ $\phi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$	(See Table 4)	

$\sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} = (a+b)^n$ $\qquad \int_0^\infty t^{n-1} e^{-t}\,dt = \Gamma(n) = (n-1)!$

$\sum_{k=0}^{\infty} ar^k = a/(1-r)$ $\qquad \sum_{k=0}^{n} ar^k = (a - ar^{n+1})/(1-r)$

$\sum_{k=0}^{\infty} x^k/k! = e^x$ $\qquad \sum_{k=0}^{n} k = n(n+1)/2$

A **and** B: $A \cap B$ $\qquad$ **not** A: A^c $\qquad$ A **or** B: $A \cup B$

$\mathsf{P}[A|B] = \mathsf{P}[A \cap B]/\mathsf{P}[B]$ $\qquad \mathsf{P}[B|A] = \mathsf{P}[A \cap B]/\mathsf{P}[A]$

$\mathsf{P}[A \cap B] = \mathsf{P}[A|B]\,\mathsf{P}[B]$ $\qquad \mathsf{P}[A \cup B] = \mathsf{P}[A] + \mathsf{P}[B] - \mathsf{P}[A \cap B]$

Independent $\Leftrightarrow \mathsf{P}[A \cap B] = \mathsf{P}[A]\,\mathsf{P}[B]$; Exclusive $\Leftrightarrow \mathsf{P}[A \cap B] = 0$

Bayes: $\mathsf{P}[A_i|B] = \frac{\mathsf{P}[B|A_i]\mathsf{P}[A_i]}{\mathsf{P}[B|A_1]\mathsf{P}[A_1] + \ldots + \mathsf{P}[B|A_k]\mathsf{P}[A_k]}$ $A_i \cap A_j = \emptyset,\quad \mathsf{P}[A_1] + \ldots + \mathsf{P}[A_k] = 1$

STA 113 Name: ____________________

Probability and Statistics in Engineering: Final Examination

	Continuous	Discrete
CDF	$F(x) = \int_{-\infty}^{x} f(t)dt$	$= \sum_{t\le x} p(t)$
pdf, pmf	$f(x) = F'(x)$	$p(x) = F(x) - F(x-)$
Mean	$\mu = \mathsf{E}[X] = \int_{-\infty}^{\infty} x\,f(x)dx$	$= \sum xp(x)$
Variance	$\sigma^2 = \mathsf{V}[X] = \mathsf{E}[(X-\mu)^2] = \mathsf{E}[X^2] - \mathsf{E}[X]^2$	$= \mathsf{E}[(X-\mu)^2] = \mathsf{E}[X^2] - \mathsf{E}[X]^2$
	$= \int_{-\infty}^{\infty} (x-\mu)^2\,f(x)dx$	$= \sum (x-\mu)^2 p(x)$
Expectation	$\mathsf{E}[g(X)] = \int_{-\infty}^{\infty} g(x)\,f(x)dx$	$= \sum_x g(x)\,p(x)$
	$\mathsf{E}[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\,f(x,y)dx\,dy$	$= \sum_x \sum_y g(x,y)\,p(x,y)$
Covariance	$\mathsf{Cov}[X,Y] = \mathsf{E}[(X-\mu_x)(Y-\mu_y)]$	$= \mathsf{E}[(X-\mu_x)(Y-\mu_y)]$
	$= \mathsf{E}[XY] - \mathsf{E}[X]\mathsf{E}[Y]$	$= \mathsf{E}[XY] - \mathsf{E}[X]\mathsf{E}[Y]$
	$= \iint_{-\infty,-\infty}^{\infty,\infty} (x-\mu_x)(y-\mu_y)\,f(x,y)dx\,dy$	$= \sum_x \sum_y (x-\mu_x)(y-\mu_y)\,p(x,y)$
Correlation	$\rho[X,Y] = \mathsf{Cov}[X,Y]/\sigma_X\sigma_Y$	
Conditional	$f_1(x\|y) = f(x,y)/f_2(y)$	$p_1(x\|y) = p(x,y)/p_2(y)$
Marginals	$f_1(x) = \int_{-\infty}^{\infty} f(x,y)dy$	$p_1(x) = \sum_y p(x,y)$
	$f_2(y) = \int_{-\infty}^{\infty} f(x,y)dx$	$p_2(y) = \sum_x p(x,y)$
Independent	$\Leftrightarrow f(x,y) = f_1(x)\,f_2(y)$	$\Leftrightarrow p(x,y) = p_1(x)\,p_2(y)$
	$\Leftrightarrow f_1(x\|y) = f_1(x)$	$\Leftrightarrow p_1(x\|y) = p_1(x)$
	$\Leftrightarrow f_2(y\|x) = f_2(y)$	$\Leftrightarrow p_2(y\|x) = p_2(y)$

Conf. Intervals	$\mu = \bar{x} \pm z_{\alpha/2} S/\sqrt{n}$	Large Sample
	$= \bar{x} \pm t_{\alpha/2} S/\sqrt{n}$	Small Sample (normal)
	$p = \hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}\,\hat{q}\,/n}$	Population Proportion
Diff. of Means	$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{s_1^2/n_1 + s_2^2/n_2}$	Large Samples
	$= (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} s_p \sqrt{1/n_1 + 1/n_2}$	Small Samples (normal, same σ^2)
	$= \overline{x_1 - x_2} \pm z_{\alpha/2} s_\Delta/\sqrt{n}$	Matched Pairs, large n
	$= \overline{x_1 - x_2} \pm t_{\alpha/2} s_\Delta/\sqrt{n}$	Matched Pairs, small n, normal
Sample Mean, Var	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
Pooled Var. Estimate	$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$	$\nu = n_1 + n_2 - 2$