STA 113 Name: ____________________________

Probability and Statistics in Engineering: First Test

2:15-3:30 pm Thursday, February 20, 1997

This is a **closed-book** examination, so please do not refer to your notes, the text, or to any other books. A few pages of formulas, tables, and blank worksheets are appended to the exam. Please do not use your own scratch paper; you may use your own calculator but may not share one. Each problem counts the same (20 points), but they are not all equally long or equally difficult; make sure you finish all the short easy ones!

Please ask me to explain anything you don't understand in the questions (wording, definitions, *etc.*). It is to your advantage to write your solutions as clearly as possible, and to show enough of your work to make clear how you got your answer (especially important if you used a calculator in some key step)—even correct answers may be awarded no credit without a clear explanation of your method. Good luck! Please sign the Duke Honor Code:

I have neither given nor received aid on this examination.

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signature

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
	/100

STA 113

Probability and Statistics in Engineering: First Test

1. The Duke/Motorolla Fabrication Plant (DMFP) has three photoetching machines for producing integrated circuits, with different capacities and different yield rates (the capacity is given as a proportion of the total DMFP production; yield rate is the fraction of wafers processed by that machine that are *not* defective):

Machine:	A	B	C
Capacity:	40%	50%	10%
Yield:	95%	98%	90%

a. (5pt) Find the proportion of defective wafers produced by the DMFP.

P[defective] = ____________

b. (10pt) Find the proportion of DMFP's defective wafers that are produced by machine C:

P[C | defective] = ____________

c. (5pt) If one of the three machines could be replaced by a machine of the same capacity that was perfect (had 100% yield), which one should be replaced, and why?

Circle one: A B C Reasoning:

Probability and Statistics in Engineering: First Test

2. A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assume that resistance follows a normal distribution.

a. (10pt) If resistance can be measured to any degree of accuracy, what percentage of resistors will have a measured resistance Ω that exceeds 43 ohms?

   $\mathsf{P}[\Omega > 43] =$ ____________

b. (10pt) If resistance can be measured only to the nearest ohm, what percentage of resistors will have a **measured** resistance Ω that exceeds 43 ohms? (Hint: Think carefully about rounding here— what must the true resistance be for its measurement to (strictly) exceed 43?)

   $\mathsf{P}[\Omega > 43] =$ ____________

Probability and Statistics in Engineering: First Test

3. Suppose that the current X drawn by Dale's toaster is a random variable with the exponential distribution with mean $\mu = 10$ amps (hence with pdf $f(x) = .1e^{-x/10}$, $x > 0$); this toaster has a resistance of $R = 12$ ohms, so the power drawn is $E = I^2R = 12 \cdot X^2$ watts. Find the expectation of the toaster's power draw:

$\mathsf{E}[12X^2] =$ ____________

Probability and Statistics in Engineering: First Test

4. Past experience has shown that 80% of the pages typed by a certain secretary (Pat) are error-free. Let Y be the number of errors on a randomly-selected page of Pat's typing.

a. (8pt) If Pat's errors follow a Poisson distribution, what is the mean number μ of errors per page, and what is the probability of exactly one error? Give numerical answers.

$\mu = \mathsf{E}[Y] =$ ____________ $\qquad$ $\mathsf{P}[Y = 1] =$ ____________

b. (8pt) If Pat's errors follow the geometric failure distribution with probability mass function $\mathsf{P}[Y = y] = pq^y$, $y = 0, 1, 2, ...$, which has mean $\mu = q/p$ and variance $\sigma^2 = q/p^2$, what is the mean number μ of errors per page, and what is the probability of exactly one error?*

$\mu = \mathsf{E}[Y] =$ ____________ $\qquad$ $\mathsf{P}[Y = 1] =$ ____________

c. (4pt) Which of these two distributions seems to you to be a better choice for modelling Pat's errors, and why?

Circle one: Poi Geo Reasoning:

* Note that our text (and this test's formula sheet) give the geometric *trial* distribution of the number $X = Y + 1$ of *trials* instead of the number Y of *failures*, with mean $\mathsf{E}[X] = 1/p = \mathsf{E}[Y] + 1 = (q/p) + 1$.

Probability and Statistics in Engineering: First Test

5. Volkswagen speedometers give exactly the correct speed *on average*, but with a normally-distributed error with standard deviation 5mph; thus the true speed of a driver whose speedometer reads s will really be a random variable X with a normal distribution with mean s and variance $5^2 = 25$. Val is driving a Volkswagon on Route I-40, where the posted speed-limit is 65mph. Val is in a hurry, but knows there is a speed trap along this route; in North Carolina, State Troopers issue tickets to all drivers exceeding the posted speed limit by at least 10mph.

a. (10pt) What is the fastest Val can try to drive (i.e., what will the *spedometer* say) and still have only a 1% chance of receiving a ticket?

b. (5pt) What is the fastest Val can try to drive and still have only a 10% chance of receiving a ticket?

c. (5pt) If Val's speedometer reads at least 5mph too low (so $S \leq X - 5$), what is the (conditional) probability that in fact it reads at least 10mph too low?

STA 113

Probability and Statistics in Engineering: First Test

Distributions

Bernoulli	$\Pr[X=x]=p^x q^{1-x},\ x=0,1$	$\mu=p,$	$\sigma^2=pq$
Binomial	$\Pr[X=x]=\binom{n}{x}p^x q^{n-x},\ x=0,1,\ldots,n$	$\mu=np,$	$\sigma^2=npq$
Poisson	$\Pr[X=x]=\lambda^x e^{-\lambda}/x!,\ x=0,1,\ldots$	$\mu=\lambda,$	$\sigma^2=\lambda$
Geometric	$\Pr[X=x]=pq^{x-1},\ x=1,2,\ldots$	$\mu=1/p,$	$\sigma^2=q/p^2$
Neg. Binomial	$\Pr[X=x]=\binom{x-1}{r-1}p^r q^{x-r},\ x=r,r+1,\ldots$	$\mu=r/p,$	$\sigma^2=rq/p^2$
Hypergeometric	$\Pr[X=x]=\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}},\ x=0,1,\ldots,n$	$\mu=\frac{nr}{N},$	$\sigma^2=$ a mess
Exponential	$\Pr[X\le x]=1-e^{-x/\beta},\ x\in(0,\infty)$	$\mu=\beta,$	$\sigma^2=\beta^2$
	$f(x)=\frac{1}{\beta}e^{-x/\beta},\ x>0;\ 0,\ x\le 0$		
Gamma	$\Pr[X\le x]=1-e^{-x/\beta}\sum_{k=0}^{\alpha-1}(x/\beta)^k/k!,\ x>0$	$\mu=\alpha\beta,$	$\sigma^2=\alpha\beta^2$
	$f(x)=\frac{x^{\alpha-1}}{\beta^\alpha\,\Gamma(\alpha)}e^{-x/\beta},\ x>0;\ 0,\ x\le 0$		
Uniform	$\Pr[X\le x]=(x-a)/(b-a),\ x\in(a,b)$	$\mu=\frac{a+b}{2},$	$\sigma^2=\frac{(b-a)^2}{12}$
	$f(x)=\frac{1}{b-a},\ a<x<b;\ 0,\ x\notin(a,b)$		
Normal	$\Pr[X\le x]=\Phi(\frac{x-\mu}{\sigma})=\begin{cases}.5+A(\frac{\lvert x-\mu\rvert}{\sigma}) & \text{if } x>\mu\\ .5-A(\frac{\lvert x-\mu\rvert}{\sigma}) & \text{if } x<\mu\end{cases}$	(See Table 4)	
	$\phi(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$		

$\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}=(a+b)^n$ $\qquad$ $\int_0^\infty t^{n-1}e^{-t}\,dt=\Gamma(n)=(n-1)!$

$\sum_{k=0}^{\infty}ar^k=a/(1-r)$ $\qquad$ $\sum_{k=0}^{n}ar^k=(a-ar^{n+1})/(1-r)$

$\sum_{k=0}^{\infty}x^k/k!=e^x$ $\qquad$ $\sum_{k=0}^{n}k=n(n+1)/2$

A **and** B: $A\cap B$ $\qquad$ **not** A: A^c $\qquad$ A **or** B: $A\cup B$

$\mathsf{P}[A|B]=\mathsf{P}[A\cap B]/\mathsf{P}[B]$ $\qquad$ $\mathsf{P}[B|A]=\mathsf{P}[A\cap B]/\mathsf{P}[A]$

$\mathsf{P}[A\cap B]=\mathsf{P}[A|B]\,\mathsf{P}[B]$ $\qquad$ $\mathsf{P}[A\cup B]=\mathsf{P}[A]+\mathsf{P}[B]-\mathsf{P}[A\cap B]$

Independent $\Leftrightarrow \mathsf{P}[A\cap B]=\mathsf{P}[A]\,\mathsf{P}[B]$; Exclusive $\Leftrightarrow \mathsf{P}[A\cap B]=0$

Bayes: $\mathsf{P}[A_i|B]=\frac{\mathsf{P}[B|A_i]\mathsf{P}[A_i]}{\mathsf{P}[B|A_1]\mathsf{P}[A_1]+\ldots+\mathsf{P}[B|A_k]\mathsf{P}[A_k]}$ $A_i\cap A_j=\emptyset,\quad \mathsf{P}[A_1]+\ldots+\mathsf{P}[A_k]=1$

STA 113

Probability and Statistics in Engineering: First Test

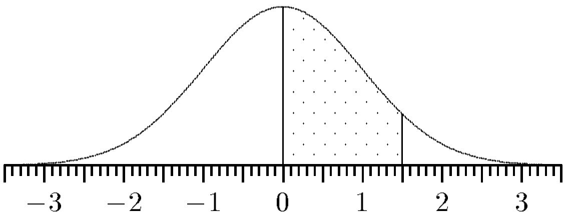
	Continuous	Discrete
CDF	$F(x) = \int_{-\infty}^{x} f(t)dt$	$= \sum_{t \le x} p(t)$
pdf, pmf	$f(x) = F'(x)$	$p(x) = F(x) - F(x-)$
Mean	$\mu = \mathsf{E}[X] = \int_{-\infty}^{\infty} x\, f(x)dx$	$= \sum xp(x)$
Variance	$\sigma^2 = \mathsf{V}[X] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$	$= \sum (x-\mu)^2 p(x)$
	$= \mathsf{E}[(X-\mu)^2]$	$= \mathsf{E}[(X-\mu)^2]$
	$= \mathsf{E}[X^2] - \mathsf{E}[X]^2$	$= \mathsf{E}[X^2] - \mathsf{E}[X]^2$
Expectation	$\mathsf{E}[g(X)] = \int_{-\infty}^{\infty} g(x)\, f(x)dx$	$= \sum_x g(x)\, p(x)$
	$\mathsf{E}[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f(x,y)dx\, dy$	$= \sum_x \sum_y g(x,y)\, p(x,y)$
Covariance	$\mathsf{Cov}[X,Y] = \iint_{-\infty,-\infty}^{\infty,\infty} (x-\mu_x)(y-\mu_y)\, f(x,y)dx\, dy$	$= \sum_x \sum_y (x-\mu_x)(y-\mu_y)\, p(x,y)$
	$= \mathsf{E}[(X-\mu_x)(Y-\mu_y)]$	$= \mathsf{E}[(X-\mu_x)(Y-\mu_y)]$
	$= \mathsf{E}[XY] - \mathsf{E}[X]\mathsf{E}[Y]$	$= \mathsf{E}[XY] - \mathsf{E}[X]\mathsf{E}[Y]$
Correlation	$\rho[X,Y] = \mathsf{Cov}[X,Y]/\sigma_X \sigma_Y$	
Conditional	$f_1(x\|y) = f(x,y)/f_2(y)$	$p_1(x\|y) = p(x,y)/p_2(y)$
Marginals	$f_1(x) = \int_{-\infty}^{\infty} f(x,y)dy$	$p_1(x) = \sum_y p(x,y)$
	$f_2(y) = \int_{-\infty}^{\infty} f(x,y)dx$	$p_2(y) = \sum_x p(x,y)$
Independent	$\Leftrightarrow f(x,y) = f_1(x)\, f_2(y)$	$\Leftrightarrow p(x,y) = p_1(x)\, p_2(y)$
	$\Leftrightarrow f_1(x\|y) = f_1(x)$	$\Leftrightarrow p_1(x\|y) = p_1(x)$
	$\Leftrightarrow f_2(y\|x) = f_2(y)$	$\Leftrightarrow p_2(y\|x) = p_2(y)$

STA 113

Probability and Statistics in Engineering: First Test

TABLE 4 Normal Curve Areas

$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE 7 Critical Values

$A(1.282) = 0.4000$ $A(1.645) = 0.4500$ $A(1.960) = 0.4750$ $A(2.326) = 0.4900$

$A(2.576) = 0.4950$ $A(3.090) = 0.4990$ $A(3.291) = 0.4995$ $A(3.719) = 0.4999$

STA 113 Name:______________________

Probability and Statistics in Engineering: First Test

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STA 113 Name: ______________________________

Probability and Statistics in Engineering: First Test

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