STA 113 Name:____________________

Probability and Statistics in Engineering: First Test

2:15-3:30 pm Thursday, February 19, 1998

This is a **closed-book** examination so please place your notes, text, and any other books on the floor. You may use a single one-sided $8\frac{1}{2} \times 11$" sheet of notes or formulas, and may use your own calculator, but may not share these materials. A normal distribution table and two blank work-sheets are appended to the exam. Each problem counts the same (20 points), but they are not all equally long or equally difficult; use your time wisely.

Numerical answers should be given to at least four correct decimal places, or as exact fractions reduced to lowest terms. Please ask me to explain anything you don't understand in the questions (wording, definitions, *etc.*). It is to your advantage to write your solutions as clearly as possible, and to show enough of your work to make clear how you got your answer (especially important if you used a calculator in some key step)—even a correct answer may be awarded zero credit without a clear explanation of your method. Good luck!

Please sign the Duke Honor Code:

I have neither given nor received unauthorized aid on this examination.

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signature

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
	/100

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1. It is known that 40 percent of patients naturally recover from a certain disease. A drug is claimed to raise the recovery rate to 80 percent of all patients with the disease. An experimental group of four randomly-selected patients with the disease are given the drug; let X denote the number of patients within this group who recover, and let P denote the fraction of all patients with the disease who will recover using the drug. Recall the Instructions (on page 0) about numerical answers.

a. What is the probability that exactly three of the four subjects recover, if the drug is worthless so the true recovery rate is 40 percent?

$\mathsf{P}[X = 3|P = 0.40] =$ ____________

b. What is the probability that exactly three of the four subjects recover, if the drug works as claimed so the true recovery rate is 80 percent?

$\mathsf{P}[X = 3|P = 0.80] =$ ____________

c. If we are uncertain about the drug's effectiveness, and believe that the recovery percentage P is equally likely to be 0.40 and 0.80, i.e., $\mathsf{P}[P = .40] = \mathsf{P}[P = .80] = 0.50$, THEN what is the probability that exactly three of the four subjects recover?

$\mathsf{P}[X = 3] =$ ____________

d. If we initially believe that the recovery percentage P is equally likely to be 0.40 and 0.80, i.e., $\mathsf{P}[P = .40] = \mathsf{P}[P = .80] = 0.50$, then perform the trial and discover that exactly three of the fours subjects recover, what is the (conditional) probability that the drug works as claimed?

$\mathsf{P}[P = 0.80|X = 3] =$ ____________

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2. For large enough n the binomial distribution of the number of "successes" in n independent trials with the same probability p of success each time is well approximated by the normal distribution with the same mean $\mu = np$ and variance $\sigma^2 = np(1-p)$ as the binomial. Let Y be the number of subjects who recover in a larger trial of the same experimental drug as in Problem 1. Use the normal approximation to give numerical answers to the questions below:

a. If the drug works as claimed so the true recovery rate is $P = 0.80$, what are the expected number $\mu = \mathsf{E}[Y]$ of subjects who would recover, and what is the variance $\sigma^2 = \mathsf{V}[Y]$, in a trial with $N = 100$ subjects?

$\mu =$ ____________ $\sigma^2 =$ ____________

b. If the drug works as claimed with true recovery rate $P = 0.80$, what is the approximate probability that at least $^3/_4$ of the subjects would recover in a trial with $N = 100$ subjects?

$\mathsf{P}[Y \geq 75 | P = 0.80] =$ ____________

c. How many subjects would we need to ensure that the probability that at least $^3/_4$ of the subjects recover is at least 99% if the drug works as claimed with true recovery rate $P = 0.80$? That is, how large must N be so that $\mathsf{P}[Y \geq .75N | P = 0.80] \geq 0.99$? Show your work!

$N \geq$ ____________

d. If initially we believe that the drug is equally likely to be worthless or effective, i.e., $\mathsf{P}[P = .40] = \mathsf{P}[P = .80] = 0.50$, and we discover that exactly three-fourths the subjects in a 100-subject trial recover, THEN what is the (conditional) probability that the drug works as claimed?

$\mathsf{P}[P = 0.80 | Y = 75] =$ ____________

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3. At each millisecond $t = 1, 2, \ldots$ patch clamps measure electrical conductivity at two points on a cell membrane, indicating whether ion channels at those locations are "open" or "closed". Let $X_t = 1$ if the ion channel at the first location is open after t milliseconds, or $X_t = 0$ if it is closed; similarly let $Y_t = 1$ if the ion channel at the second location is open. Assume (perhaps unrealistically) that all the measurements at both locations are independent, and that at all times $\mathsf{P}[X_t = 1] = .2$ and $\mathsf{P}[Y_t = 1] = .4$.

a. What is the probability that at least 10 milliseconds go by before the first channel is open, i.e., find

$\mathsf{P}[X_1 = 0, \ldots, X_{10} = 0] =$ ____________

b. What is the probability that the two channels are in the same state at some fixed time t?

$\mathsf{P}[X_t = Y_t] =$ ____________

c. What is the probability that the first channel is open at least half of the first four measurements?

$\mathsf{P}[X_t = 1$ for at least twice in $t = 1..4] =$ ____________

d. Why might it be unrealistic to assume independence?

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4. For large n and small p the binomial distribution of the number of "successes" in n independent trials all with the same probability p of success is well approximated by the Poisson distribution with the same mean $\lambda = np$; we discovered this in class as we computed the probability of catching a specific number of fish in t hours, a large number of seconds.

Suppose that a memory chip has 256000 memory cells, each with the same probability $p = 10^{-6}$ of being defective, and suppose that defects are independent. Use the Poisson approximation to find numerical answers (correct to four decimals) to the following questions about the number X of defective cells on a randomly selected chip (note that "successes" here are *defective* cells):

a. Find the probability that a chip will have zero defects

$P[X = 0] = \underline{\hspace{3cm}}$

b. The chip foundary has discovered a way to build in redundancy by adding a few "spare" cells and circuitry to switch over any defective cell with one of the spares. What is the probability that a chip will still be fully functional if two spare cells are available?

$\mathsf{P}[X \le 2] = \underline{\hspace{3cm}}$

c. What fraction of defective chips will be made fully functional using this technology? This is a question about a conditional probability concerning X; please fill in all three blanks:

$\mathsf{P}[X \underline{\hspace{2cm}} \mid X \underline{\hspace{2cm}}] = \underline{\hspace{3cm}}$

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5. Let X be the length of time (measured in hours) you spend waiting at the airport for your highschool sweetheart to arrive for Spring Break. Parking at RDU airport costs \$.75 for each hour or fraction of an hour; let Y be X, rounded up to the next higher integer (so $Y = 4$ when $X = \pi$, for example). Assume that X has the exponential distribution with mean $\mathsf{E}[X] = 0.50$.

a. Which does X have, a probability density function (pdf) or a probability mass function (pmf)? Check one of these below, and give the value (which we'll call $f_1(x)$) at every real number $-\infty < x < \infty$.

Pick one: ◯ pdf ◯ pmf $f_1(x) =$ ____________

b. Which does Y have, a probability density function (pdf) or a probability mass function (pmf)? Check one of these below, and give the value (which we'll call $f_2(y)$) at every real number $-\infty < y < \infty$.

Pick one: ◯ pdf ◯ pmf $f_2(y) =$ ____________

c. Do you recognize the distribution of Y? What is its name, what is/are the parameter name(s) and value(s), and what is the mean $\mu = \mathsf{E}[Y]$?

Name: ____________ Parameter(s): ____________ Mean: $\mu =$ _____

d. What is the probability you pay less than one dollar for parking? Why?

Done!

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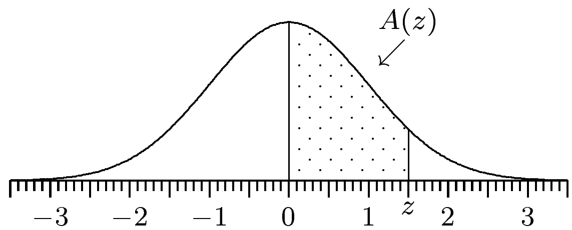
TABLE 4 Normal Curve Areas

$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE 7 Critical Values

$A(1.282) = 0.4000$ $A(1.645) = 0.4500$ $A(1.960) = 0.4750$ $A(2.326) = 0.4900$

$A(2.576) = 0.4950$ $A(3.090) = 0.4990$ $A(3.291) = 0.4995$ $A(3.719) = 0.4999$

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