STA 113 Name:______________________________

Probability and Statistics in Engineering: Second Test

2:15-3:30 pm Thursday, April 2, 1998

This is a closed-book exam so please do not refer to your notes, the text, or to any other books (please put them on the floor). You may use a single sheet of notes or formulas and a calculator. Normal z and Student t tables and blank worksheets are attached to the exam.

You must show your work to get partial credit. Even correct answers will not receive full credit without justification. Please give all numerical answers to at least four correct digits or as exact fractions reduced to lowest terms. Write your solutions as clearly as possible and make sure it's easy to find your answers (circle them if necessary), since you will not receive credit for work that I cannot understand or find. Good Luck!

Please sign the Duke Honor Code:
I have neither given nor received unauthorized aid on this examination.

____
signature

1.	/20
2.	/20
3.	/20
4.	/20
5.	/20
	/100

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1. The concentrations X, Y of carbon monoxide at two urban locations at a randomly chosen time follow a joint probability distribution with density function

$$f(x,y) = \begin{cases} c(x^2 + 4xy), & 0 \le x \le 1,\, 0 \le y \le 1 \\ 0, & \text{elsewhere.} \end{cases}$$

a. (3pt) Find the value of c. Show your reasoning.

$c =$ ____________

b. (8pt) Find the marginal PDF and CDF for X. Be sure your answers are correct for *all* values of $-\infty < x < \infty$.

$f_1(x) =$ ____________

$F_1(x) =$ ____________

c. (5pt) Find the conditional density of Y, given $X = x$, for each $0 < x < 1$.

$f_{2|1}(y|x) =$ ____________

d. (4pt) Are X and Y independent? Why?

Circle one: Yes No Reasoning:

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2. Calculus is taught at Duke in two different formats: MTH 31/32, a traditional lecture course, and MTH 31L/32L, a laboratory course sequence developed at Duke. Most MTH 32L students are continuing from MTH 31L; most MTH 32 students are freshmen who placed out of MTH 31 with the AP exam but did not have a lab-based first-semester course.

To assess how well the laboratory approach compares with the more traditional approach, six second-semester students are selected randomly from a lecture based MTH 32 section and another six from a laboratory based MTH 32L section and their performances are compared on a standard calculus test, with the following results:

Lec: $X = 79,\ 85,\ 64,\ 82,\ 80,\ 90$	$\bar{X} = 80$	$\sum(X-80)^2 = 386$
Lab: $Y = 86,\ 92,\ 82,\ 72,\ 86,\ 92$	$\bar{Y} = 85$	$\sum(Y-85)^2 = 278$

a. (12pt) Perhaps students in the two classes perform equally well, or perhaps those in the laboratory class perform better. Test an appropriate hypothesis and present your conclusions (you must state H_0 and H_A, select a test statistic, choose a level α, compute the approximate P-value, and report your conclusion).

b. (4pt) State at least two assumptions you made above.

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c. (4pt) Even if students' performance *is* better in the laboratory class, this may not indicate that the laboratory approach is superior to the lecture approach. Explain why.

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3. Recall the calculus test data from Problem 2:

$$\begin{array}{llll} \text{Lec: } X = 79,\ 85,\ 64,\ 82,\ 80,\ 90 & \bar{X} = 80 & \sum(X-80)^2 = 386 \\ \text{Lab: } Y = 86,\ 92,\ 82,\ 72,\ 86,\ 92 & \bar{Y} = 85 & \sum(Y-85)^2 = 278 \end{array}$$

a. (10pt) Give a 90% confidence interval for the mean performance of students in the laboratory-based class.

b. (10pt) Give a 90% confidence interval for the difference in mean performances of students in the two classes.

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4. IBM expects that at least 90% of the microprocessors they produce using a new experimental copper-trace fabrication process would meet specifications; in a test run of $n = 4$ processors, they found two defective ones.

Is this strong enough evidence to conclude that their yield is really less than 90%? Find the exact significance level (P-value) and test the hypothesis at level $\alpha = .05$. Be sure to state your hypothesis and its alternative clearly (4pts), give your test statistic (2pts) and its distribution under H_0 (4pts), and report both P-value (6pts) and conclusion (4pts) clearly.

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5. (20pt) Mark the single best answer for each question (5pt each):

a. Why do we sometimes use the t distribution with some number ν of degrees of freedom, instead of the normal distribution?

- ☐ Because we're not sure if the population distribution is normal or not.
- ☐ Because we don't know the population variance σ^2 and must estimate it from the sample by s^2.
- ☐ Because the sample is smaller than 30 or so, even if we do know σ^2.
- ☐ Because the population variances σ_1^2 and σ_2^2 might not be the same.

b. When do we use "matched pair" hypothesis tests and confidence intervals?

- ☐ When we have two populations, both normally distributed, but maybe with different variances.
- ☐ When we have two samples with the same sample size.
- ☐ When we are testing a two-sided alternative hypothesis.
- ☐ When we have observations for both populations from each of several experimental subjects, and want to minimize the effect of variations among the subjects.

c. If we estimate a normal mean by $\mu = \bar{Y} \pm 2.0\, s/\sqrt{n}$, with $n = 6$, what is the (approximate) probability that the interval we find will contain μ?

☐ 80%	☐ 90%	☐ 95%	☐ 97.5%	
☐ 98%	☐ 99%	☐ 99.8%	☐ 99.9%	☐ Can't tell

d. In a test of the null hypothesis $H_0 : \mu = 2$ with a normally distributed population we find $z = \frac{\bar{X}-2}{\sigma/\sqrt{n}} = 1.8$. Give the P-value and decision (accept or reject) for each of the following alternatives, using level $\alpha = .05$.

$H_a : \mu \neq 2$	☐ Reject H_0	☐ Can't reject H_0	$P =$ ________
$H_a : \mu > 2$	☐ Reject H_0	☐ Can't reject H_0	$P =$ ________
$H_a : \mu < 2$	☐ Reject H_0	☐ Can't reject H_0	$P =$ ________

 Done!

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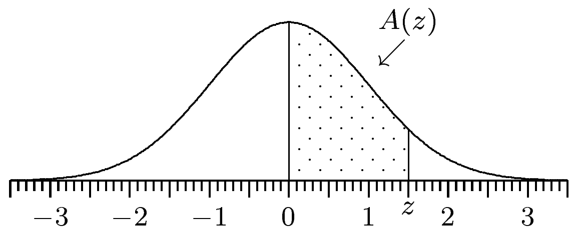
TABLE 4 Normal Curve Areas

$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE 7 Critical Values

$A(1.282) = 0.4000$ $A(1.645) = 0.4500$ $A(1.960) = 0.4750$ $A(2.326) = 0.4900$

$A(2.576) = 0.4950$ $A(3.090) = 0.4990$ $A(3.291) = 0.4995$ $A(3.719) = 0.4999$

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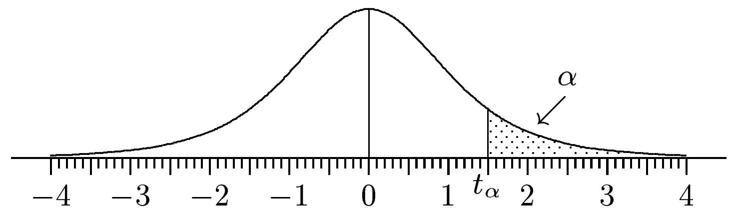
TABLE 7 Critical Values for Student's t

$$\alpha = \int_{t_\alpha}^{\infty} \frac{c\ dt}{(1 + t^2/\nu)^{(\nu+1)/2}}$$



ν	$t_{.40}$	$t_{.30}$	$t_{.20}$	$t_{.15}$	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$	$t_{.0001}$
1	0.325	0.727	1.376	1.9626	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.3862	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.2498	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.1896	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.1558	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.1342	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.1192	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.1081	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.0997	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.0931	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.0877	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.0832	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.0795	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.0763	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.0735	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.0711	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.0690	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.0672	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.0655	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.0640	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.0627	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.0614	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.0603	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.0593	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.0584	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.0575	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.0567	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.0560	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.0553	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.0547	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.0500	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.0455	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.0409	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.0364	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

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