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Distributions

Bernoulli	$\Pr[X=x]=p^x q^{1-x},\ x=0,1$	$\mu=p,$	$\sigma^2=pq$
Binomial	$\Pr[X=x]=\binom{n}{x}p^x q^{n-x},\ x=0,1,\ldots,n$	$\mu=np,$	$\sigma^2=npq$
Poisson	$\Pr[X=x]=\lambda^x e^{-\lambda}/x!,\ x=0,1,\ldots$	$\mu=\lambda,$	$\sigma^2=\lambda$
Geometric	$\Pr[X=x]=pq^{x-1},\ x=1,2,\ldots$	$\mu=1/p,$	$\sigma^2=q/p^2$
Neg. Binomial	$\Pr[X=x]=\binom{x-1}{r-1}p^r q^{x-r},\ x=r,r{+}1,\ldots$	$\mu=r/p,$	$\sigma^2=rq/p^2$
Hypergeometric	$\Pr[X=x]=\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}},\ x=0,1,\ldots,n$	$\mu=\frac{nr}{N},$	$\sigma^2=$ a mess
Exponential	$\Pr[X\le x]=1-e^{-x/\beta},\ x\in(0,\infty)$	$\mu=\beta,$	$\sigma^2=\beta^2$
	$f(x)=\frac{1}{\beta}e^{-x/\beta},\ x>0;\ 0,\ x\le 0$		
Gamma	$\Pr[X\le x]=1-e^{-x/\beta}\sum_{k=0}^{\alpha-1}(x/\beta)^k/k!,\ x>0$	$\mu=\alpha\beta,$	$\sigma^2=\alpha\beta^2$
	$f(x)=\frac{x^{\alpha-1}}{\beta^\alpha\,\Gamma(\alpha)}e^{-x/\beta},\ x>0;\ 0,\ x\le 0$		
Uniform	$\Pr[X\le x]=(x-a)/(b-a),\ x\in(a,b)$	$\mu=\frac{a+b}{2},$	$\sigma^2=\frac{(b-a)^2}{12}$
	$f(x)=\frac{1}{b-a},\ a<x<b;\ 0,\ x\notin(a,b)$		
Normal	$\Pr[X\le x]=\Phi(\frac{x-\mu}{\sigma})=\begin{cases}.5+A(\frac{\lvert x-\mu\rvert}{\sigma}) & \text{if } x>\mu\\ .5-A(\frac{\lvert x-\mu\rvert}{\sigma}) & \text{if } x<\mu\end{cases}$	(See Table 4)	
	$\phi(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$		

$\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}=(a+b)^n$ $\qquad$ $\int_0^\infty t^{n-1}e^{-t}\,dt=\Gamma(n)=(n-1)!$

$\sum_{k=0}^{\infty}ar^k=a/(1-r)$ $\qquad$ $\sum_{k=0}^{n}ar^k=(a-ar^{n+1})/(1-r)$

$\sum_{k=0}^{\infty}x^k/k!=e^x$ $\qquad$ $\sum_{k=0}^{n}k=n(n+1)/2$

A **and** B: $A\cap B$ $\qquad$ **not** A: A^c $\qquad$ A **or** B: $A\cup B$

$\mathsf{P}[A|B]=\mathsf{P}[A\cap B]/\mathsf{P}[B]$ $\qquad$ $\mathsf{P}[B|A]=\mathsf{P}[A\cap B]/\mathsf{P}[A]$

$\mathsf{P}[A\cap B]=\mathsf{P}[A|B]\,\mathsf{P}[B]$ $\qquad$ $\mathsf{P}[A\cup B]=\mathsf{P}[A]+\mathsf{P}[B]-\mathsf{P}[A\cap B]$

Independent $\Leftrightarrow \mathsf{P}[A\cap B]=\mathsf{P}[A]\,\mathsf{P}[B]$; Exclusive $\Leftrightarrow \mathsf{P}[A\cap B]=0$

Bayes: $\mathsf{P}[A_i|B]=\frac{\mathsf{P}[B|A_i]\mathsf{P}[A_i]}{\mathsf{P}[B|A_1]\mathsf{P}[A_1]+\ldots+\mathsf{P}[B|A_k]\mathsf{P}[A_k]}$ $A_i\cap A_j=\emptyset,\quad \mathsf{P}[A_1]+\ldots+\mathsf{P}[A_k]=1$

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	Continuous	Discrete
CDF	$F(x) = \int_{-\infty}^{x} f(t)dt$	$= \sum_{t \le x} p(t)$
pdf, pmf	$f(x) = F'(x)$	$p(x) = F(x) - F(x-)$
Mean	$\mu = \mathsf{E}[X] = \int_{-\infty}^{\infty} x\, f(x)dx$	$= \sum xp(x)$
Variance	$\sigma^2 = \mathsf{V}[X] = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)dx$	$= \sum (x-\mu)^2 p(x)$
	$= \mathsf{E}[(X-\mu)^2] = \mathsf{E}[X^2] - \mathsf{E}[X]^2$	$= \mathsf{E}[(X-\mu)^2] = \mathsf{E}[X^2] - \mathsf{E}[X]^2$
Expectation	$\mathsf{E}[g(X)] = \int_{-\infty}^{\infty} g(x)\, f(x)dx$	$= \sum_x g(x)\, p(x)$
	$\mathsf{E}[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, f(x,y)dx\, dy$	$= \sum_x \sum_y g(x,y)\, p(x,y)$
Covariance	$\mathsf{Cov}[X,Y] = \iint_{-\infty,-\infty}^{\infty,\infty} (x-\mu_x)(y-\mu_y)\, f(x,y)dx\, dy$	$= \sum_x \sum_y (x-\mu_x)(y-\mu_y)\, p(x,y)$
	$= \mathsf{E}[(X-\mu_x)(Y-\mu_y)]$	$= \mathsf{E}[(X-\mu_x)(Y-\mu_y)]$
	$= \mathsf{E}[XY] - \mathsf{E}[X]\mathsf{E}[Y]$	$= \mathsf{E}[XY] - \mathsf{E}[X]\mathsf{E}[Y]$
Correlation	$\rho[X,Y] = \mathsf{Cov}[X,Y]/\sigma_X\sigma_Y$	
Conditional	$f_1(x\|y) = f(x,y)/f_2(y)$	$p_1(x\|y) = p(x,y)/p_2(y)$
Marginals	$f_1(x) = \int_{-\infty}^{\infty} f(x,y)dy$	$p_1(x) = \sum_y p(x,y)$
	$f_2(y) = \int_{-\infty}^{\infty} f(x,y)dx$	$p_2(y) = \sum_x p(x,y)$
Independent	$\Leftrightarrow f(x,y) = f_1(x)\, f_2(y)$	$\Leftrightarrow p(x,y) = p_1(x)\, p_2(y)$
	$\Leftrightarrow f_1(x\|y) = f_1(x)$	$\Leftrightarrow p_1(x\|y) = p_1(x)$
	$\Leftrightarrow f_2(y\|x) = f_2(y)$	$\Leftrightarrow p_2(y\|x) = p_2(y)$

Conf. Intervals	$\mu = \bar{x} \pm z_{\alpha/2} S/\sqrt{n}$	Large Sample
	$= \bar{x} \pm t_{\alpha/2} S/\sqrt{n}$	Small Sample (normal)
	$p = \hat{p} \pm z_{\alpha/2}\sqrt{\hat{p}\,\hat{q}\,/n}$	Population Proportion
Diff. of Means	$\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{s_1^2/n_1 + s_2^2/n_2}$	Large Samples
	$= (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2} s_p \sqrt{1/n_1 + 1/n_2}$	Small Samples (normal, same σ^2)
	$= \overline{x_1 - x_2} \pm z_{\alpha/2} s_\Delta/\sqrt{n}$	Matched Pairs, large n
	$= \overline{x_1 - x_2} \pm t_{\alpha/2} s_\Delta/\sqrt{n}$	Matched Pairs, small n, normal
Sample Mean, Var	$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$
Pooled Var. Estimate	$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$	$\nu = n_1 + n_2 - 2$

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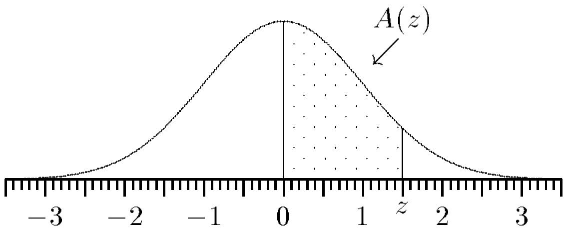
TABLE 4 Normal Curve Areas

$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt:$$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

TABLE 7 Critical Values

$A(1.282) = 0.4000$ $A(1.645) = 0.4500$ $A(1.960) = 0.4750$ $A(2.326) = 0.4900$

$A(2.576) = 0.4950$ $A(3.090) = 0.4990$ $A(3.291) = 0.4995$ $A(3.719) = 0.4999$

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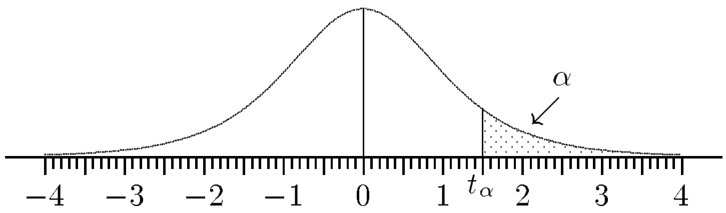
TABLE 7 **Critical Values for Student's t**

$$\alpha = \int_{t_\alpha}^{\infty} \frac{c\,dt}{(1+t^2/\nu)^{(\nu+1)/2}}$$



ν	$t_{.40}$	$t_{.30}$	$t_{.20}$	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$	$t_{.0001}$
1	0.325	0.727	1.376	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719