Answers for Homework 2

1. Pr(a subject has adverse reaction)= $\frac{1}{25} = 0.04$,

This is a binomial distribution with parameter $p = 0.04$, $n = 10$: $X \sim Bin(p = 0.04, n = 10)$

a) Exactly two:

$Pr(X = 2) = \binom{10}{2}(0.04)^2(0.96)^8 = 0.0519$

b) No more than one:

$Pr(X = 0 or 1) = Pr(0)+Pr(1) = \binom{10}{0}(0.04)^0(0.96)^10+\binom{10}{0}(0.04)^1(0.96)^9 = 0.9418$

c) No

$Pr(X = 0) = \binom{10}{0}(0.04)^0(0.96)^10 = 0.6648$

d) At least two

$Pr(X \geq 2) = 1 - Pr(0) - Pr(1) = 1 - 0.9418$(the answer in b.)$= 0.0582$

e) At least two but no more than four

$Pr(2 \leq X \leq 4) = Pr(2) + Pr(3) + Pr(4) = \binom{10}{2}(0.04)^2(0.96)^8 + \binom{10}{3}(0.04)^3(0.96)^7+\binom{10}{4}(0.04)^4(0.96)^6 = 0.0519+0.0058+0.0004 = 0.0581$

2. The number of deaths per unit per year $X \sim Poisson(m = 0.7)$,

$Pr(X = k) = \frac{0.7^k}{k!}e^{-0.7}$, $k = 0, 1, 2, 3, ...$

a)

$Pr(x = 0) = \frac{0.7^0}{0!}e^{-0.7} = 0.4966$

b)

Each unit-year has a probability of 0.4966 to report 0 deaths. 280 unit-years in all. It is a binomial distribution $X \sim Bin(p = 0.4966, n = 280)$, success event defined as reporting 0 deaths. The approximate number of records with 0 deaths, mathematically put, the expected number of successes in 280 trials, is

$E(X) = n * p = 280 * 0.4966 \approx 139$.

c)

As a property of Poisson distributions, the expectation always equals the parameter, the expectation for this Poisson($m = 0.7$) is 0.7. So the expected number of deaths per unit-year is 0.7. For all 280 unit-years, the expected total number of deaths is

$280 * 0.7 = 196$

3. Way 1.

25 cigars, randomly choose 3. In order not to get fired, must choose these 3 from the unloaded 20 cigars. So the probability is

$$\frac{\binom{20}{3}}{\binom{25}{3}}$$

Way2.

Denote the event "the k-th chosen cigar is firecracker free" as "k-U"

Pr(Not fired)= $Pr(1U\&2U\&3U) = Pr(1U)Pr(2U|1U)Pr(3U|1U\&2U)$

where,

$Pr(1U) = \frac{20}{25}$;

$Pr(2U|1U) = \frac{19}{24}$;

$Pr(3U|1U\&2U) = \frac{18}{23}$;

Way 3.

The textbook way, hypergeometric distribution (p.179), $N = 25, n = 3, r = 5, y = 0$

All these ways will give the same answer: $\frac{20}{25}\frac{19}{24}\frac{18}{23} = 0.496$

4. Similar to problem 1., $X \sim Bin(p = 0.4, n = 3)$

a) 0.064

b) 0.432

c) 0.648

Optional $X, Y \sim Bin(n = 2, p = 0.5)$, $Z \sim Poisson(\lambda = 1)$

The possible values of X include 0,1,2. So the event $[X = Y]$ is equivalent to the union of $[X = Y = 0]$ and $[X = Y = 1]$ and $[X = Y = 2]$. Thus,

$$
\begin{aligned}
&Pr(X = Y) \\
&= Pr(X = Y = 0) + Pr(X = Y = 1) + Pr(X = Y = 2) \\
&= Pr(X = 0)Pr(Y = 0) + Pr(X = 1)Pr(Y = 1) + Pr(X = 2)Pr(Y = 2) \\
&= (\binom{2}{0}(0.5)^0(0.5)^2)^2 + (\binom{2}{1}(0.5)^1(0.5)^1)^2 + (\binom{2}{2}(0.5)^2(0.5)^0)^2 \\
&= 0.375
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&Pr(X = Z) \\
&= \sum_{k=0}^{2} Pr(X = k)Pr(Z = K) \\
&= \sum_{k=0}^{2} \binom{2}{k} 0.5^2 \frac{1^k}{k!} e^{-1} \\
&= 0.322
\end{aligned}
$$

So, $Pr(X = Y) > Pr(X = Z)$, that is, the event $X = Y$ is more likely.