12-11

a. For the given data,

$$\begin{aligned}\beta &= b \pm t_{.025}\,\frac{s}{\sqrt{\sum x^2}} \\ &= 800 \pm 2.02\,\frac{7300}{\sqrt{900}} \\ &= 800 \pm 491.5 \\ & \quad 308.5 < \beta < 1291.5\end{aligned}$$

Note that, since $n = 50$, the number of degrees of freedom of the t distribution is 48. This critical point can not be found in the tables, so we used d.f.=40, having in mind that this way we are being conservative and therefore we get a larger interval. Linear interpolation is also OK.

b. Yes, since $0 \notin (308.5, 1291.5)$.

d. Prediction interval:

$$\begin{aligned}Y_0 &= a + b\,X_0 \pm t_{.025}\;s\,\sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X}_0)^2}{\sum x^2} + 1} \\ &= 1200 + 800 \cdot 10 \pm 2.02 \cdot 7300 \cdot \sqrt{\frac{1}{50} + \frac{(10-11)^2}{900} + 1} \\ &= 9200 \pm 14900.8\end{aligned}$$

d. No cause-effect relation between annual income and level of education can be inferred from his analysis, since the data arises from an uncontrolled observational study. Uncontrolled confounding variables can be responsible for the observed relation.

12-13

Since $t = b/SE$ and $SE = .27/t_{.025}$, we get $t = 2.76$. Also the hypotheses is $H_0 : \beta \leq 0$, so $0.0025 < p - value < 0.005$. Note that we used a normal standard table since the sample size is large: $n = 1000$.

Extra Exercise

a) We have $n = 32$, so that $df = 30$ and $t_{.025} = 2.05$.

$$\beta = b \pm t_{.025}\;SE$$

$$
\begin{aligned}
&= 102.289 \pm 2.05 \cdot 24.23 \\
&= 102.289 \pm 49.43 \\
&\quad 52.86 < \beta < 151.72
\end{aligned}
$$

$$
\begin{aligned}
\alpha &= a \pm t_{.025}\, SE \\
&= -6553.57 \pm 2.04 \cdot 1661.96 \\
&= -6553.57 \pm 3390.40 \\
&\quad -9943.97 < \alpha < -3163.17
\end{aligned}
$$

b) Yes, since the 95% confidence interval for β does not contain zero.

c) The hypotheses to test is $H_0 :\ \beta = 100$, so the t statistic is

$$t = \frac{b - 100}{SE} = \frac{102.289 - 100}{24.23} = 0.094.$$

We want to calculate a two-sided p-value, so, looking at the table, we recognize that $p - value > 2 \cdot 0.25$, that is, the p-value is larger than 50%.

d) Looking at the output tables, we see that $s^2 = 18761.24$.