STA 110B "solutions" **Homework 12**

13-8

a. Note that $df = 66 \Leftrightarrow 3 \Leftrightarrow 1 = 62 \approx 60$, so $t_{.025} = 2$.

X_1

95% CI $= 0.021 \pm 2 \cdot 0.019 = 0.021 \pm 0.38$
t-ratio $= 0.021/0.019 = 1.105$
p-value $> 2 \cdot 0.1 = 0.2$ (two-sided p-value)

X_2

95% CI $= 0.075 \pm 2 \cdot 0.034 = 0.075 \pm 0.068$
t-ratio $= 0.075/0.034 = 2.206$
p-value $2 \cdot 0.010 \Leftrightarrow\Leftrightarrow 2 \cdot 0.025$, ie, $0.02 \Leftrightarrow\Leftrightarrow 0.05$ (two-sided p-value)

X_3

95% CI $= 0.043 \pm 2 \cdot 0.018 = 0.043 \pm 0.0.036$
t-ratio $= 0.043/0.018 = 2.390$
p-value $\approx 2 \cdot 0.010 = 0.02$ (two-sided p-value)

b. We are assuming that the 66 students are a random sample for a hypothetical large population, which in this case in not so reasonable – they are very far from having been selected at random, they are just the available students at a certain time.

c. The variable X_3, because it has the largest t-ratio and smallest p-value.

d. We should keep the first regressor first because there is enough statistical evidence to support that, and second because it is very reasonable to expect that a student's rank (from the bottom) is positively related with his/her score in a test.

13-11

a. $\Delta Y = b_2 \Delta X_2 = \Leftrightarrow 1.1(5 \Leftrightarrow 2) = \Leftrightarrow 3.3$, therefore the answer should be less \$3.3 per front foot.

b. $\Delta Y = b_3 \Delta X_3 = \Leftrightarrow 1.34(\Leftrightarrow 1/2) = 0.67$, and we conclude that the price per front foot is \$0.67 higher if the lot is 1/2 a mile closer to the nearest paved road (other things being equal).

c. $\Delta Y = b_1 \Delta X_1 = 1.5(5 \Leftrightarrow 1) = 6$, so the price is \$6 per front foot higher than in 1970. (Same kind of lot: size and distance from the nearest paved road.)

13-17

$$
\begin{aligned}
Y &= c_1 + 3.38X_1 + 0.0364X_2 \\
X_1 &= c_2 \Leftrightarrow 0.0725X_2 \\
X_2 &= c_3 \Leftrightarrow 4.33X_1
\end{aligned}
$$

a. 3.38 (coefficient of X_1 in the multiple regression)

b. $3.38 + (\Leftrightarrow 4.33) \cdot 0.0364 = 3.222$

c. It would increase the expected yield by 3.38 units per year, if the temperature is maintained fixed.

d. The simple regression coefficient is equal to the total relation, so the answer is 3.222 in the first case and $0.0364 + (\Leftrightarrow 0.0725) \cdot 3.38 = \Leftrightarrow 0.209$ in the second.

14-3

a. $df = 1072 \Leftrightarrow 5 \Leftrightarrow 1 = 1066 \Rightarrow t_{.025} \approx z_{.025} = 1.96$

AGE	$\Leftrightarrow 3.9 \pm 1.96 \times 1.8 = \Leftrightarrow 3.9 \pm 3.53$
SMOK	$\Leftrightarrow 9.0 \pm 1.96 \times 2.2 = \Leftrightarrow 9.0 \pm 4.31$
CHEMW	$\Leftrightarrow 350 \pm 1.96 \times 46 = \Leftrightarrow 350 \pm 90.16$
FARMW	$\Leftrightarrow 380 \pm 1.96 \times 53 = \Leftrightarrow 380 \pm 103.88$
FIREW	$\Leftrightarrow 180 \pm 1.96 \times 54 = \Leftrightarrow 180 \pm 105.84$

b. age and amount of cigarettes per day; 9; lower.

c. 30; higher.

d. 39; lower.

e. $20 \times 9 = 180$; lower.

d. $180/39 \approx 4.6$ years; important variables may be omited.

14-9

a. **i.** $D_A = D_B = 0$, so $\hat{Y} = 61$.

ii. $D_A = 1,\ D_B = 0$, so $\hat{Y} = 61 + 9 = 70$.

iii. $D_A = 0,\ D_B = 1$, so $\hat{Y} = 61 + 12 = 73$.

b. $\bar{C}$ = answer **i.**, $\bar{A}$ = answer **ii.** and $\bar{B}$ = answer **iii.**.

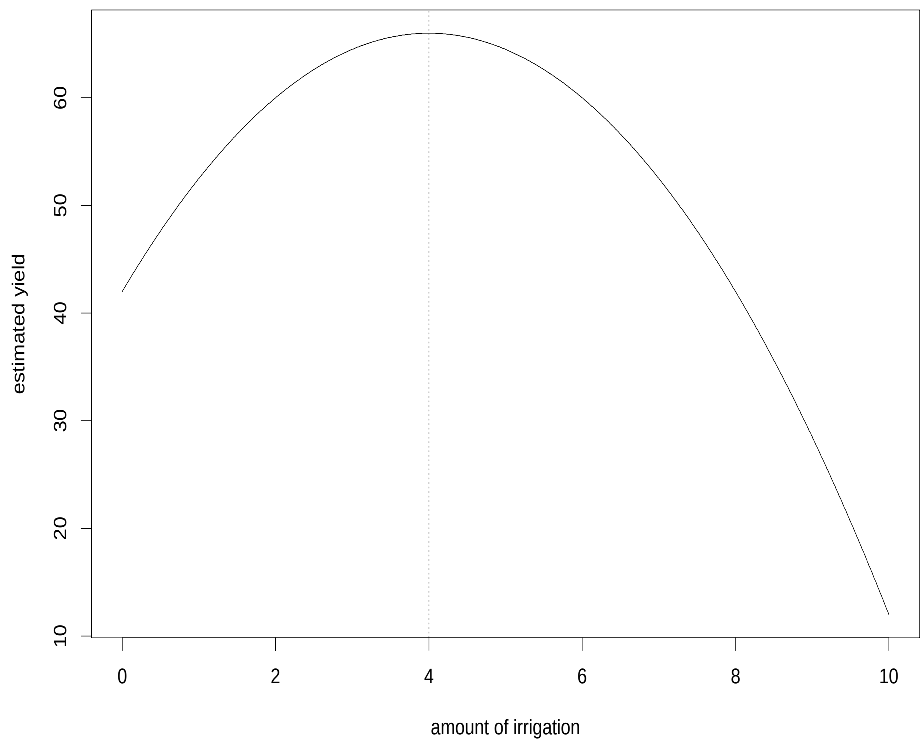
c. one factor ANOVA; "Yield" is the response, the factor is the type of fertilizer — A, B or none (C).

14-13

a. Not very credible, since too much water should have a negative effect on the yield.

b. We should test that the coefficient for I^2 is zero. $df = 14 \Leftrightarrow 2 \Leftrightarrow 1 = 11$ and the t ratio is $\Leftrightarrow 1.5/.4 = \Leftrightarrow 3.75$ which implies that the p-value is between $2 \times .001$ and $2 \times .0025$, that is $0.002 < p \Leftrightarrow value < 0.005$. This conveys very little evidence in support of this hypothesis, which confirms our previous answer.

c. The maximum occurs at $I = 4$, but we can use less water without decreasing too much the yield.



d. $\Delta \hat{Y} = 12 \cdot \Delta I \Leftrightarrow 1.5 \cdot \Delta I^2 = 12 \cdot (3 \Leftrightarrow 2) \Leftrightarrow 1.5 \cdot (3^2 \Leftrightarrow 2^2) = 4.5$

14-18

a. elasticity $= 1.3$ (slope)

b. relative change in price $\approx$ change in log price, so if price increased by 3%, the price changed 0.03. Hence, $\Delta \log Q = 1.3 \times 0.03 = 0.039$ and, for the same reason, we conclude that Q increased by 3.9%

c. relative change in Q of 10% $\approx$ change in $\log Q$ of 0.1, so,

$$0.1 = 1.3\Delta \log P \Leftrightarrow \Delta \log P = \frac{0.1}{1.3} = 0.007$$

so that the relative change in price should be approximately 7.7%.