Name:

STAT 113 – Midterm 2
Practice Exam

1a. [2pt] Suppose y is normally distributed random variable with mean $\mu = 5.0$ and variance $\sigma^2 = 4.0$, i.e. $y \sim N(5.0, 4.0)$. Find $P(3.0 < y < 12.0)$.

1b. [2pt] Suppose y is a χ^2 distributed random variable with $\nu = 12$ degrees of freedom. Find cutoffs c and d, such that $P(c < y < d) = 0.99$.

1c. [2pt] Suppose y is a Student t distributed random variable with $\nu = 10$ degrees of freedom. Find $P(-1.372 < y < 1.372)$.

1d. [2pt] Let x and y have the joint density

$$f(x,y) = \begin{cases} c\,e^{-(x+y)} & \text{if } 0 \le x < \infty; 0 \le y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

Are x and y independent? (why/why not?)

1e. [2pt] Suppose y is a χ^2 random variable with $\nu_1 = 10$ degrees of freedom, and x is a χ^2 random variable with $\nu = 7$ degrees of freedom. Assume x and y are independent. Consider $w = x + y$. Find $P(w < 10)$.

2. A discharge (or response) rate of auditory nerve fibers [recorded as the number of spkes per 200 milliseconds (ms) of noise burst] is used to measure the effect of acoustic stimuli in the auditory nerve. An empricial study of auditory nerve fiber response rates in cats resulted in a mean of 15 spikes/ms (*Journal of the Acoustical Society of America*, Feb 1986). Let y represent the auditory nerve fiber response rate for a randomly selected cat in the study.

2a. [4pt] If y is approximately a Poisson random variable, find the mean and standard deviation of y.

2b. [4pt] Assuming y is Poisson, what is sthe approximate probability that y exceeds 27 spikes/ms.

2c. [2pt] In the study, the variance of y was found to be "substantially smaller" than 15 spikes/ms. Is it reasonable to expect y to follow a Poisson process? How will this aaffect the probability computed in 2b?

3. Let x and y have the joint density

$$f(x,y) = \begin{cases} x + cy & \text{if } 1 \le x \le 2; 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

where c is a constant.

3a [3pts] Find the value of c that makes $f(x,y)$ a probability density function.

3b [3pts] Find the marginal density $f(y)$ for y.

3c [4pts] Find $f(x|y)$, the conditinal density for x given y.

3d [1pt] Are x and y independent? (why/why not?)

Name: Section:

STAT 113 – Midterm 2
Problem 4

On the actual midterm you will find the following instructions. For the practice exam, of course ignore these instructions. In particular, group work is ok.

> **Take home problem.** Tear this page off the exam, and submit by Tuesday 5pm in my office (219 Old Chemistry). There will be a file pocket on the board left of my office door).
> *No group work on this problem.*

4. The Parks and Recreation Department of a city considers a lake suitable for swimming only if the mean concentration of a particular pollutant is bellow 30ppm (parts per million). A sample of 25 specimens is obtained and analyzed for its pollutant concentration:

```
26.32 23.48 25.09 30.26 26.56 28.53 27.72 26.22 33.99 20.57 28.08 28.91
29.48 25.93 18.92 26.04 31.32 26.81 31.00 25.48 27.08 32.56 32.33 22.45
39.00
```

The data are available on the STA 113 homepage (click "exams"). Or copy the data set by typing from your acpub account:

```
cp /afs/acpub/project/sta215/parks.data parks.data
```

4a. [2pt] Using MINITAB (or any other statistics program of your choice) construct a relative frequency histogram for the data.

4b. [2pt] Find sample mean $\bar{x}$ and sample variance s^2.

4c. [4pt] Assuming that the distribution of measurements in the specimens has a mean pollutant concentration of $\mu = 30ppm$ with a standard deviation of $\sigma = 5ppm$, sketch the sampling distribution of the sample mean $\bar{x}$:

```
  _
f(x)

  ^
  |
  |
  |
  |
  |
  |
  |
  +----------+----------+---------+----------+----------+->   _
 20         25         30        35         40         45    x
```

4d. [2pt] Still assuming $\mu = 30$ and $\sigma = 5$, find a cutoff c, such that $P(\bar{x} < c) = 0.05$.

4e. [1pt] The agency will use the following rule: If $\bar{x} < c$ then the lake is declared "safe" and allow the use of the lake for swimming[1] Given the observed $\bar{x}$, should the lake be opened for swimming?

4d*. [2pt] If the standard deviation σ was not known, how would you propose to change the rule? Why?

---

[1]This type of "rule" is called a hypothesis test. There is only an $\alpha = 5\%$ chance of opening the lake when in fact it is dangerous. The rule is called a test at level $\alpha = 5\%$ of the two competing hypotheses $H_0 : \mu = 30$ vs. $H_1 : \mu < 30$.