

Name:

STAT 113 – Midterm 2
Practice Exam

- 1a. [2pt] Suppose y is normally distributed random variable with mean $\mu = 5.0$ and variance $\sigma^2 = 4.0$, i.e. $y \sim N(5.0, 4.0)$. Find $P(3.0 < y < 12.0)$.
- 1b. [2pt] Suppose y is a χ^2 distributed random variable with $\nu = 12$ degrees of freedom. Find cutoffs c and d , such that $P(c < y < d) = 0.99$.
- 1c. [2pt] Suppose y is a Student t distributed random variable with $\nu = 10$ degrees of freedom. Find $P(-1.372 < y < 1.372)$.
- 1d. [2pt] Let x and y have the joint density

$$f(x, y) = \begin{cases} c e^{-(x+y)} & \text{if } 0 \leq x < \infty; 0 \leq y < \infty \\ 0 & \text{elsewhere} \end{cases}$$

Are x and y independent? (why/why not?)

- 1e. [2pt] Suppose y is a χ^2 random variable with $\nu_1 = 10$ degrees of freedom, and x is a χ^2 random variable with $\nu = 7$ degrees of freedom. Assume x and y are independent. Consider $w = x + y$. Find $P(w < 10)$.
2. A discharge (or response) rate of auditory nerve fibers [recorded as the number of spikes per 200 milliseconds (ms) of noise burst] is used to measure the effect of acoustic stimuli in the auditory nerve. An empirical study of auditory nerve fiber response rates in cats resulted in a mean of 15 spikes/ms (*Journal of the Acoustical Society of America*, Feb 1986). Let y represent the auditory nerve fiber response rate for a randomly selected cat in the study.
- 2a. [4pt] If y is approximately a Poisson random variable, find the mean and standard deviation of y .
- 2b. [4pt] Assuming y is Poisson, what is the approximate probability that y exceeds 27 spikes/ms.
- 2c. [2pt] In the study, the variance of y was found to be “substantially smaller” than 15 spikes/ms. Is it reasonable to expect y to follow a Poisson process? How will this affect the probability computed in 2b?

3. Let x and y have the joint density

$$f(x, y) = \begin{cases} x + cy & \text{if } 1 \leq x \leq 2; 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

where c is a constant.

- 3a [3pts] Find the value of c that makes $f(x, y)$ a probability density function.
- 3b [3pts] Find the marginal density $f(y)$ for y .
- 3c [4pts] Find $f(x|y)$, the conditional density for x given y .
- 3d [1pt] Are x and y independent? (why/why not?)

STAT 113 – Midterm 2

Problem 4

On the actual midterm you will find the following instructions. For the practice exam, of course ignore these instructions. In particular, group work is ok.

Take home problem. Tear this page off the exam, and submit by Tuesday 5pm in my office (219 Old Chemistry). There will be a file pocket on the board left of my office door).
No group work on this problem.

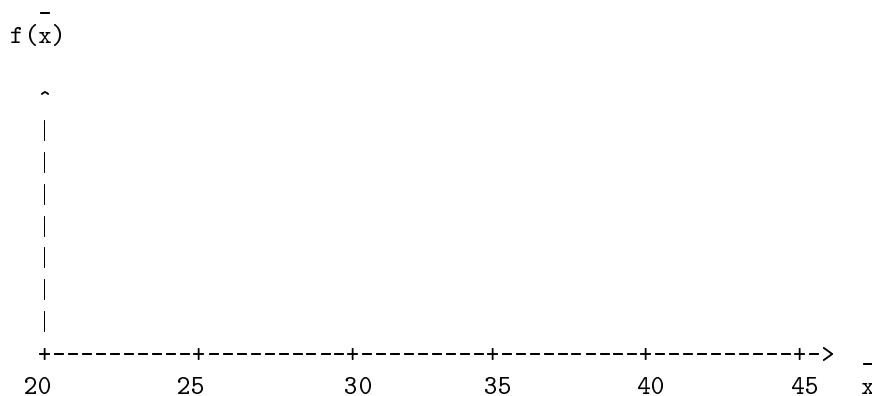
4. The Parks and Recreation Department of a city considers a lake suitable for swimming only if the mean concentration of a particular pollutant is bellow 30ppm (parts per million). A sample of 25 specimens is obtained and analyzed for its pollutant concentration:

26.32 23.48 25.09 30.26 26.56 28.53 27.72 26.22 33.99 20.57 28.08 28.91
 29.48 25.93 18.92 26.04 31.32 26.81 31.00 25.48 27.08 32.56 32.33 22.45
 39.00

The data are available on the STA 113 homepage (click “exams”). Or copy the data set by typing from your acpub account:

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cp /afs/acpub/project/sta215/parks.data parks.data
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- 4a. [2pt] Using MINITAB (or any other statistics program of your choice) construct a relative frequency histogram for the data.
- 4b. [2pt] Find sample mean $\bar{x}$ and sample variance s^2 .
- 4c. [4pt] Assuming that the distribution of measurements in the specimens has a mean pollutant concentration of $\mu = 30ppm$ with a standard deviation of $\sigma = 5ppm$, sketch the sampling distribution of the sample mean $\bar{x}$:



- 4d. [2pt] Still assuming $\mu = 30$ and $\sigma = 5$, find a cutoff c , such that $P(\bar{x} < c) = 0.05$.
- 4e. [1pt] The agency will use the following rule: If $\bar{x} < c$ then the lake is declared “safe” and allow the use of the lake for swimming¹ Given the observed $\bar{x}$, should the lake be opened for swimming?
- 4d*. [2pt] If the standard deviation σ was not known, how would you propose to change the rule? Why?

¹This type of “rule” is called a hypothesis test. There is only an $\alpha = 5\%$ chance of opening the lake when in fact it is dangerous. The rule is called a test at level $\alpha = 5\%$ of the two competing hypotheses $H_0 : \mu = 30$ vs. $H_1 : \mu < 30$.